

**onesteel** reinforcing **SEMINARS** Sept. 2001

**Advanced Design™ for  
Slab Deflections**

**Dr Paul Berry  
Dr Mark Patrick**

**Centre for Construction Technology and Research  
University of Western Sydney**

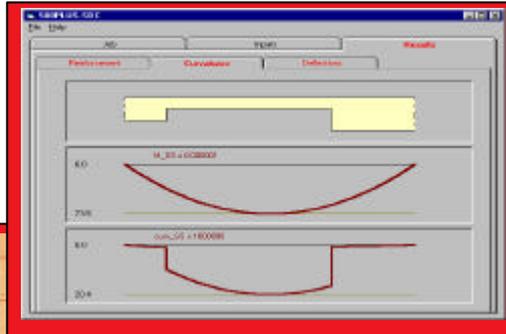
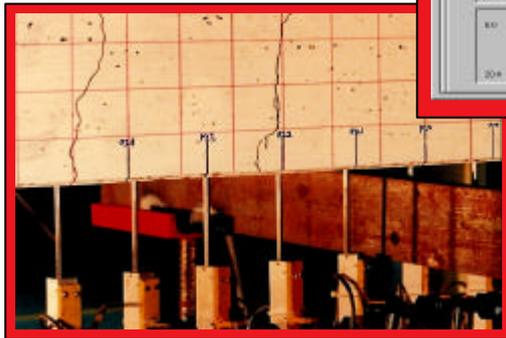
**CSTR** **onesteel** reinforcing

The Centre for Construction Technology and Research at the University of Western Sydney is progressively developing new design rules for OneSteel Reinforcing's *Guide to Reinforced Concrete Design*.

Top-tier design rules that meet the requirements for design by refined calculation defined in AS 3600-2001 are being released under the trademark of Advanced Design™. This presentation concerns rules being developed for advanced design of reinforced-concrete slabs for vertical deflection. The design rules are proprietary and should only be used when OneSteel Reinforcing's products are specified.

## Advanced Design™ for Slab Deflections

New Design Rules  
&  
Software



BETTER  
CONTROL

CSI

onesteel  
SOFTWARE

The advanced design rules have been incorporated in a new software package called 500PLUS-SDC, “SDC” standing for “slab deflection control” which will be demonstrated during the presentation.

The new design rules and software will provide the user with better control of deflections. It will be shown that more economical reinforced-concrete slabs will result. The new rules are being developed from the results of new research and tests being undertaken at the University of Western Sydney.

## Advanced Design™ for Slab Deflections

### Overview

- **AS 3600–2001 Design Tiers (One-Way Slabs)**
- **Section Analysis**
  - AS 3600–2001 Simplified Calculation
  - Advanced Design™ Options
  - Comparisons
- **Member Analysis**
  - AS 3600–2001 Simplified Calculation
  - Advanced Design™ Options
  - Comparisons



This presentation will address the design of one-way slabs. Theories are also being developed to predict the deflections of two-way slabs which will form the basis of new design design rules to be released in the future.

The design tiers available in AS 3600-2001 will be briefly described. Slab deflection design by “simplified calculation” will be briefly discussed and used in comparisons with designs obtained using the rules for various Advanced Design™ options presented.

The presentation consists of two main parts: *section analysis* when the section properties of the slab must be determined; and *member analysis* when load distribution and support conditions must also be considered and the maximum deflections calculated.

## AS 3600–2001 Design Tiers (One-Way Slabs)

### Clause 9.3 Deflection of Slabs

- **Cl. 9.3.2 *Refined calculation***
  - shrinkage and creep
  - loading history
  - cracking and tension stiffening
- **Cl. 9.3.3 *Simplified calculation***
  - equivalent beam  $\Rightarrow$  Cl. 8.5.3 *Beam deflection*
- **Cl. 9.3.4 *Deemed to comply span-to-depth ratio***
  - very limited in application



Critical issues to consider when undertaking refined calculation of one-way slabs are shrinkage and creep, loading history (e.g. when propping is removed), and flexural cracking and tension stiffening. Attention will be given to the calculation of short-term deflections in this presentation.

Simplified calculation of deflections of one-way slabs subjected to uniformly distributed loads must be performed in accordance with Clause 9.3.3 of AS 3600-2001. This Clause refers to Clause 8.5.3 and the slab must be treated as an equivalent beam.

Design of one-way slabs according to Clause 9.3.4 of AS 3600-2001 using deemed to comply span-to-depth ratio is much more conservative than using advanced design, and is also very limited in its application. Therefore it is not discussed here.

## Section Analysis



Some significant changes have been made to AS 3600-1994 concerning the calculation of short-term section properties. These changes are discussed first.

## AS 3600–2001 Simplified Calculation

### Cl. 8.5.3 Beam Deflection by Simplified Calculation

#### Cl. 8.5.3.1 *Short-term deflection*

- $I_{ef}$  by Branson's Eqn, which includes  $M_{cr}^3$
- $M_{cr}$  now reduced by shrinkage-induced tensile stress,  $f_{cs}$ , i.e.  $M_{cr} = Z (f'_{cf} - f_{cs})$

Therefore,  $I_{ef}$  is reduced:

- Deflections may affect designs more
- *Benefits of 500MPa steel are less apparent*



The cracking moment  $M_{cr}$  has been redefined in Clause 8.5.3.1. It is now necessary for designers to take account of shrinkage-induced tensile stresses that develop in the concrete which is restrained by bonded reinforcement.

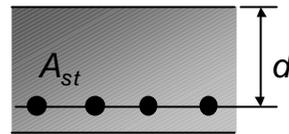
Consequently,  $M_{cr}$  can reduce significantly, and therefore  $I_{ef}$ . Deflections may increase and deflections affect designs more. The benefits of the new higher-strength 500 MPa reinforcing steel are less apparent if the design for deflection influences the area of tensile steel.

## AS 3600–2001 Simplified Calculation

### Approximation for Singly Reinforced Sections

- $f_{cs} = \frac{1.5\rho}{1 + 50\rho} E_s \varepsilon_{cs}$

$$\rho = A_{st} / (bd)$$



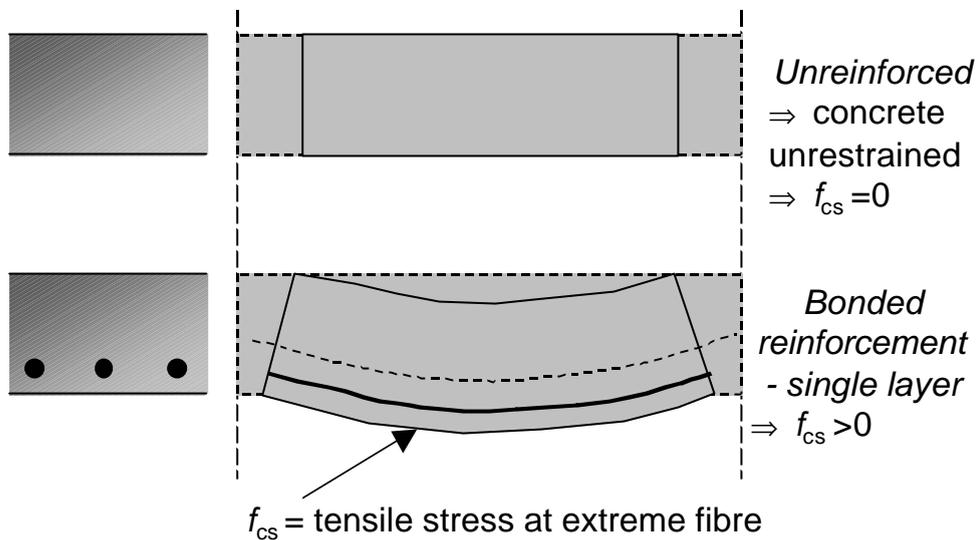
- **Basis:**

- uncracked rectangular section
- long-term stiffness ( $n = 16.7$ )
- $\varepsilon_{cs}$  is long-term (free) shrinkage strain (at 30 years)
- 40% of  $\varepsilon_{cs}$  has occurred at the time of cracking

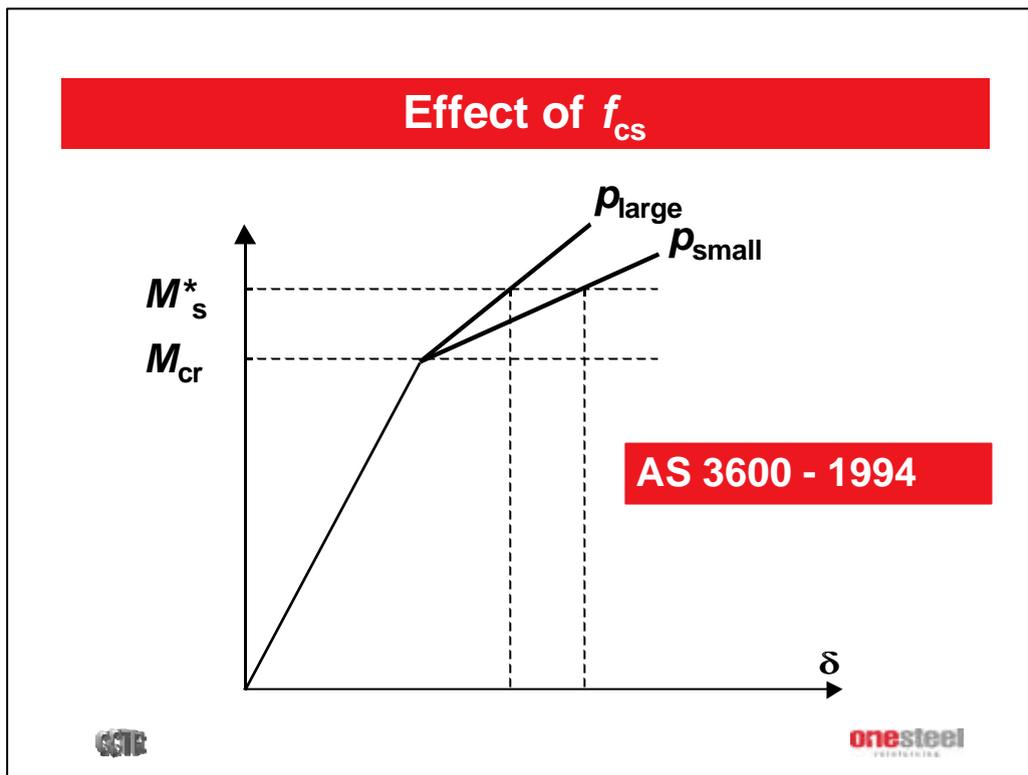


This new equation for  $f_{cs}$  is given in AS 3600-2001, but only applies to rectangular sections such as a solid slab with a single layer of reinforcement. It has been derived using elastic theory by making a number of simplifying assumptions. Designers are not given advice about how to calculate  $f_{cs}$  for other more complex situations.

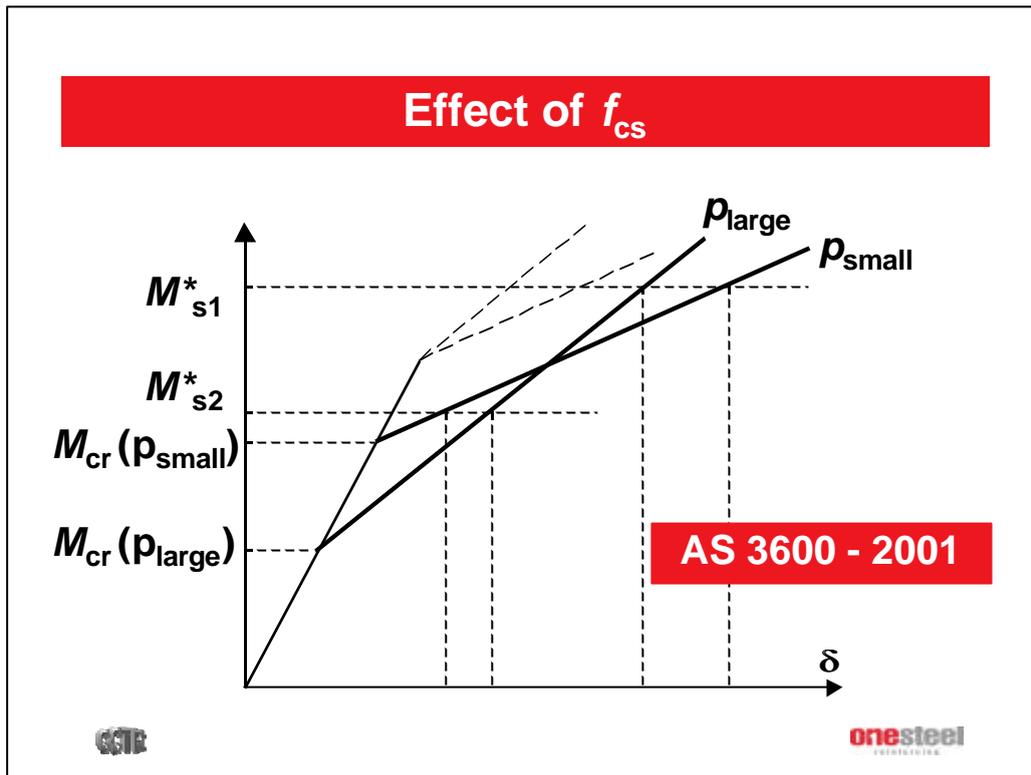
## AS 3600–2001 Simplified Calculation



This slide shows that bonded reinforcement placed asymmetrically in a solid section gives rise to a tensile stress  $f_{cs}$  which is a maximum at the extreme fibre adjacent to the reinforcement. Ignoring the effect of external restraints, curvature is also induced which causes “shrinkage warping” deflections. However these are normally ignored in design.



When designers used AS 3600-1994 they ignored the effect of shrinkage-induced tension. It was clear that at the onset of cracking larger vertical deflections would be predicted if the reinforcement ratio was reduced.



The situation is now less clear when using AS 3600-2001. Now the effect of reducing the reinforcement ratio can cause larger or smaller deflections depending on the value of the serviceability moment.

## Advanced Design™ Options

### **Option S1**

**Same assumptions, but include effect of compressive reinforcement**

### **Option S2**

**Cross-section analysis**

### **Option S3**

**Use consistent values of  $n$  and  $e_{cs}$  that both correspond to time of cracking (and don't just assume 40% of  $e_{cs}$  occurs)**



Three Advanced Design™ options S1, S2 and S3 for section analysis are being developed. They are of increasing complexity to formulate (S1 being the simplest improvement), but this will not concern the designer who will be able to use 500PLUS-SDC.

Option S1 involves the beneficial effect of compressive reinforcement when calculating  $f_{cs}$  for a solid rectangular section. In fact, in the software the formulation will be generalised such that the effect of an additional layer of reinforcement (either in tension or compression prior to cracking) can be determined. The same simplifying assumptions about the stiffness of the concrete and the free strain of the concrete at the time of cracking, as were made to derive the formula for  $f_{cs}$  in AS 3600-2001, are used.

Option S2 is similar to Option S1 except that the sections can have a general shape (e.g. voided slab) and multiple layers of reinforcement.

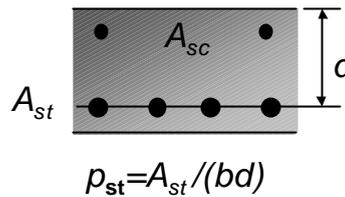
Option S3 involves a more accurate assessment of the stiffness and free shrinkage strain of the concrete. The analysis of general sections will be possible.

## Advanced Design™ - Option S1

### Include Effect of Compressive Reinforcement

- Can show that:

$$f_{cs} = \frac{1.5\rho_{st} - 0.5\rho_{sc}}{1 + 50\rho_{st}} E_s \varepsilon_{cs}$$

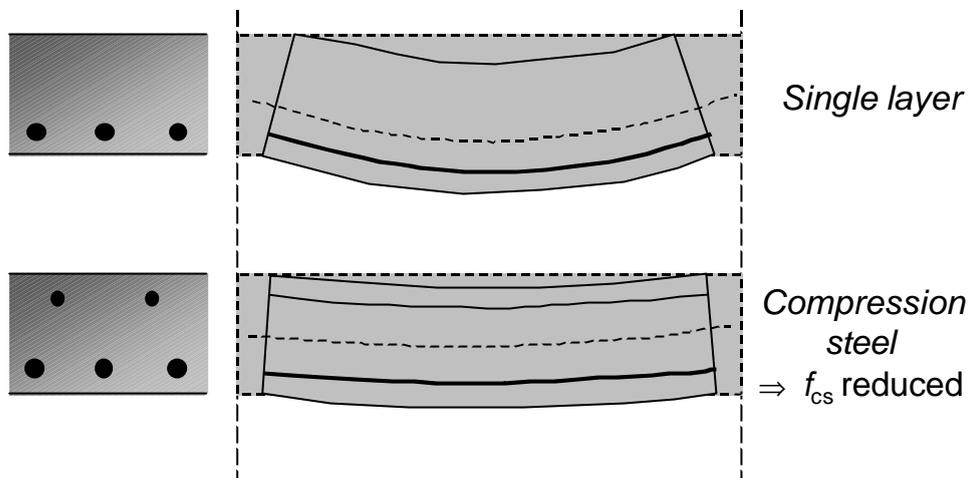


- **Assumptions:**
  - same as AS 3600–2001



It can be seen that  $f_{cs}$  is reduced by taking into account compressive reinforcement.

## Advanced Design™ - Option S1 .. /...



This slide qualitatively shows that shrinkage warping and  $f_{cs}$  are reduced by reducing the asymmetry of the reinforcement layout.

## Advanced Design™ - Option S2

### Cross-Section Analysis

- $$f_{cs} = \left[ \frac{1}{A_c} - \frac{1}{A_{tr}} + \frac{(\bar{y}_{tr} - \bar{y}_c)(D - \bar{y}_{tr})}{I_{tr}} \right] A_c E_{ef} \varepsilon_{sh}$$

where

$A_c$  = the area of the concrete alone;

$\bar{y}_c$  = the centroid of  $A_c$  ;

$A_{tr}$  = the transformed area based on  $n$ ;

$\bar{y}_{tr}$  = the centroid of  $A_{tr}$  ;

$I_{tr}$  = the second moment of area of  $A_{tr}$  ;



General geometries can be handled using this formula.

## Advanced Design™ - Option S2 .. /...

...which, in keeping with AS 3600–2001  
Simplified Calculation, gives rise to:

$$f_{cs} = 0.4 \left[ \frac{1}{A_c} - \frac{1}{A_{tr}} + \frac{(\bar{y}_{tr} - \bar{y}_c)(D - \bar{y}_{tr})}{I_{tr}} \right] A_c E_{ef} \varepsilon_{cs}$$

- **Assumptions :**
  - same as AS 3600–2001 Simplified Calculation



## Advanced Design™ - Option S3

### Use Consistent Values of $n$ and $e_{cs}$

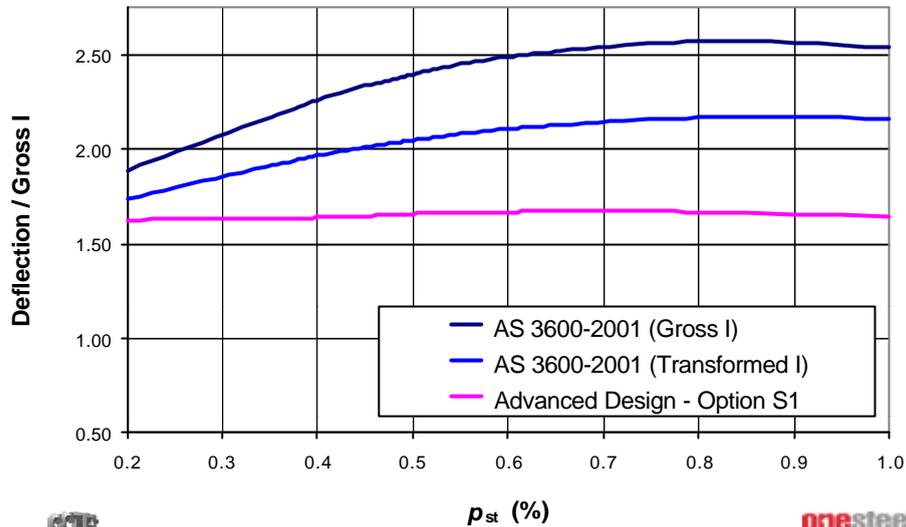
- **Short-term analysis**

- $\varepsilon_{sh} = \varepsilon_{sh}(t)$ ,  $t$  = time at cracking
- section properties based on  $n = n(t)$
- concrete modulus  $E_{ef} = E_s / n$
- tensile strength of concrete



## Comparisons

**Simply-supported slab,  $Q = 3 \text{ kPa}$ ,  $p_{sc} = p_{st}$**



In this graph, the deflections have been divided by the corresponding deflection calculated using  $I = \text{Gross } I = (bD^3)/12$ .

The slab is simply-supported with a solid, rectangular section. There is equal top and bottom steel such that  $p_{sc} = p_{st}$ . The live load  $Q$  is assumed to remain constant independent of the amount of reinforcement, which is not entirely practical, but nevertheless the example serves to illustrate some important features of the different approaches to calculate  $I_{ef}$ . The value of  $I_{ef}$  was calculated using  $M^*_{s_2}$  at the mid-span cross-section in accordance with Clause 8.5.3.1 of AS 3600-2001.

Results are shown for  $p_{st}$  down to 0.002 which is enough to satisfy the minimum bending strength requirement in Clause 8.1.4.1 of AS 3600-2001. However, it may not be enough for flexural crack control (Clause 9.4.1) or crack control for shrinkage and temperature effects (Clause 9.4.3).

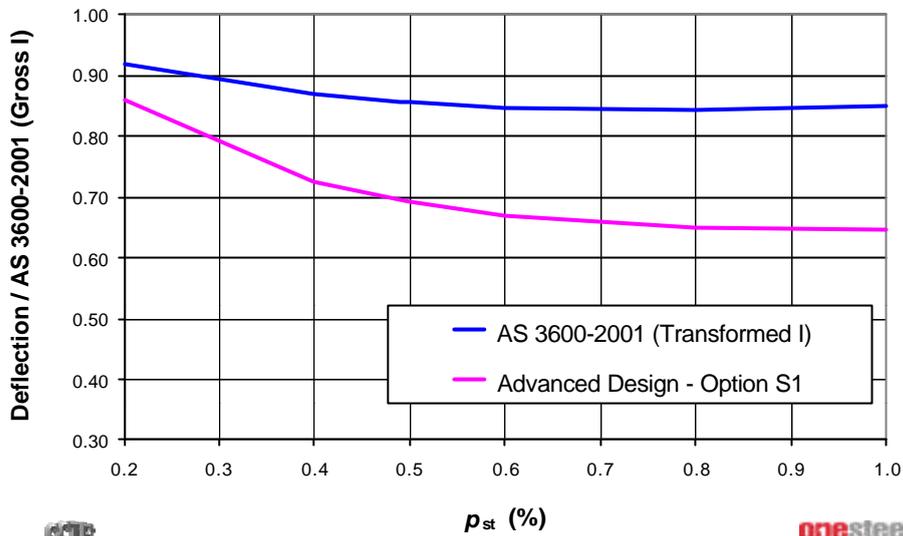
The top line has been calculated ignoring the presence of the main (longitudinal) reinforcing steel in the uncracked (gross) section. The effect of shrinkage-induced tensile stress has been included which explains why the line slopes upwards rather than downwards for smaller values of  $p_{st}$ , i.e. situation for  $M^*_{s_2}$  in Slide 10.

The middle line shows the significant benefit that can be gained if designers take account of the main reinforcement in the uncracked state.

The bottom line shows the further benefit that arises from taking into account the compressive reinforcement when calculating the second moment of area of the gross section and  $f_{cs}$ . In this case the theory predicts that  $I_{ef}$  is effectively independent of  $p_{st}$ .

## Comparisons

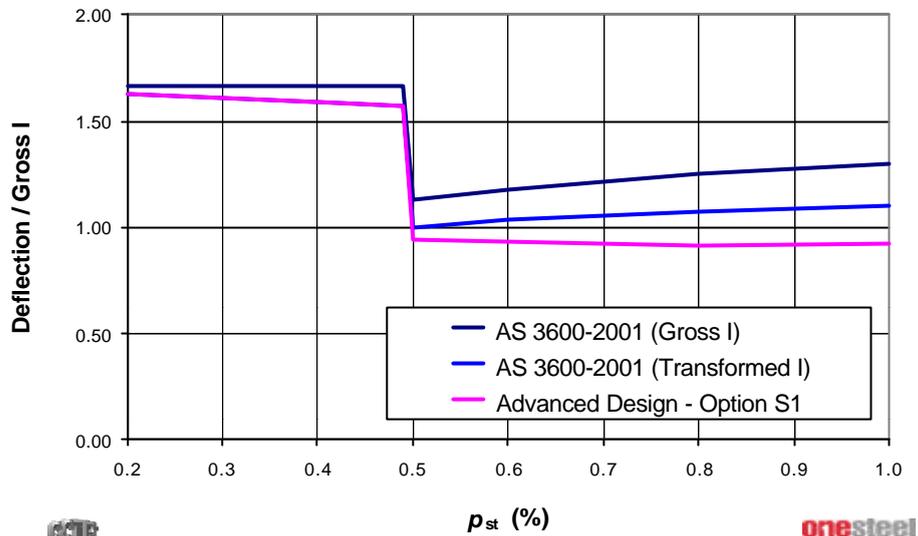
Simply-supported slab,  $Q = 3 \text{ kPa}$ ,  $\rho_{sc} = \rho_{st}$



In this graph, the deflections have been divided by the corresponding deflection determined by simplified calculation using AS 3600-2001 (Gross I), which is shown as the top line in the previous slide. It shows that for  $\rho_{st} > 0.005$ , reductions of about 15% and 30% respectively can be expected by (a) taking into account the presence of the main steel when calculating  $I$ , and (b) further including the effect of the compressive reinforcement when calculating  $f_{cs}$  and therefore  $M_{cr}$ .

## Comparisons

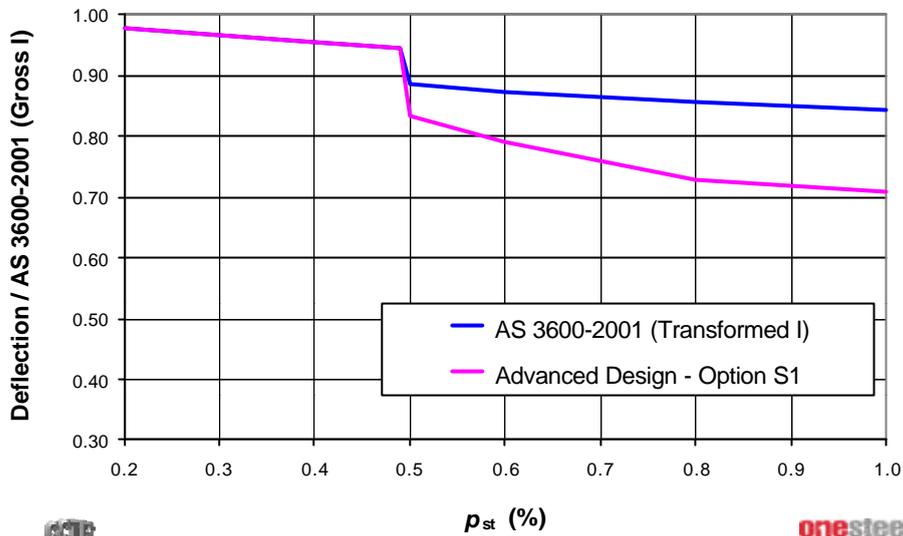
Continuous slab,  $Q = 3 \text{ kPa}$ ,  $p_{sc} = p_{st}$



A similar example is given here for a continuous slab. The discontinuity at 0.5% reinforcement ratio is due to the new upper limit placed on the effective second moment of area,  $I_{e,max}$ , of  $0.6I$  for  $p_{st} < 0.005$  in accordance with Clause 8.5.3.1 of AS 3600-2001.

## Comparisons

Continuous slab,  $Q = 3 \text{ kPa}$ ,  $\rho_{sc} = \rho_{st}$



In this graph, once again the deflections have been divided by the corresponding deflection determined by simplified calculation using AS 3600-2001 (Gross I), which is shown as the top line in the previous slide. It shows that for  $\rho_{st} > 0.005$ , reductions of at least 10% and 20% respectively can be expected by (a) taking into account the presence of the main steel when calculating  $I$ , and (b) further including the effect of the compressive reinforcement when calculating  $f_{cs}$  and therefore  $M_{cr}$ .

## Member Analysis



Opportunities for improving member analysis are now considered.

## AS 3600–2001 Simplified Calculation

### Cl. 8.5.3 Beam Deflection by Simplified Calculation

#### Cl. 8.5.3.1 *Short-term deflection*

- **Approximate uniform  $I_{ef}$  determined from values at nominated cross-sections:**
  - *Simply supported:* equal to mid-span value
  - *Internal span:* half mid-span plus quarter each support
  - *End span:* half mid-span plus half continuous support



The rules for calculating the approximate uniform value of  $I_{ef}$  for simply-supported or continuous beams and one-way slabs, as given in Clause 8.5.3.1 of AS 3600-2001, are stated here.

## Advanced Design™ Options

### **Option M1**

$I_{ef}$  calculated using Branson's modified equation for individual cross-sections with a term  $(M_{cr}/M_s^*)^4$  instead of  $(M_{cr}/M_s^*)^3$ , and integrated to obtain deflections

### **Option M2**

Iterative procedure whereby moments are recalculated using updated stiffnesses

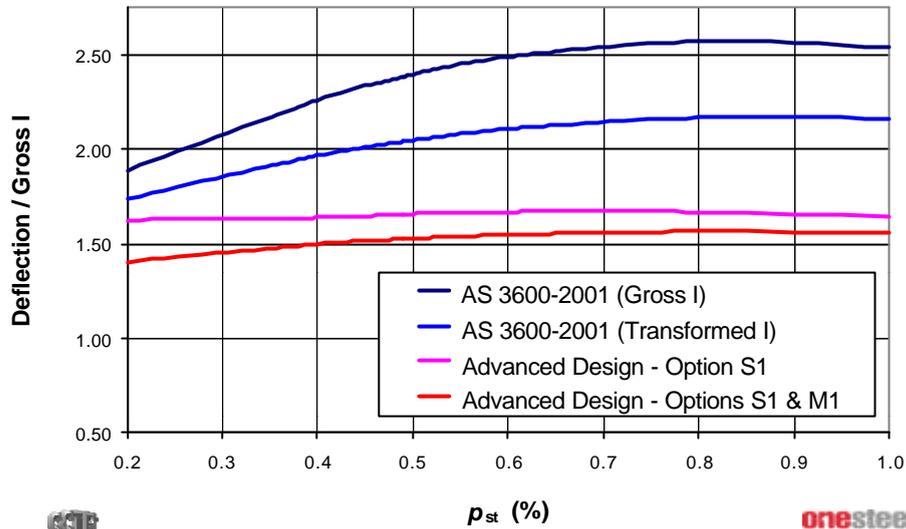


More general procedures that more accurately account for variations in the effective second moment of area  $I_{ef}$  along a slab span are needed. Two options M1 and M2 are currently being investigated, option M1 being the simpler which is therefore of primary interest to this presentation.

When using Option M1, the designer must ensure compatibility of end slopes between adjacent spans by adjusting the end moments as necessary.

## Comparisons

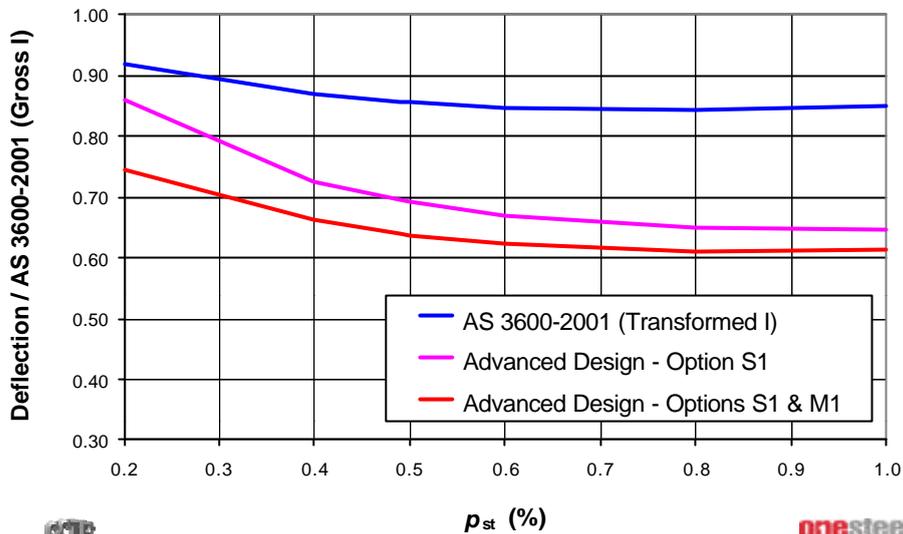
Simply-supported slab,  $Q = 3 \text{ kPa}$ ,  $p_{sc} = p_{st}$



This graph is the same as that shown in Slide 17 except that the new bottom line has been added which shows the benefit in using option M1 when performing the member analysis.

## Comparisons

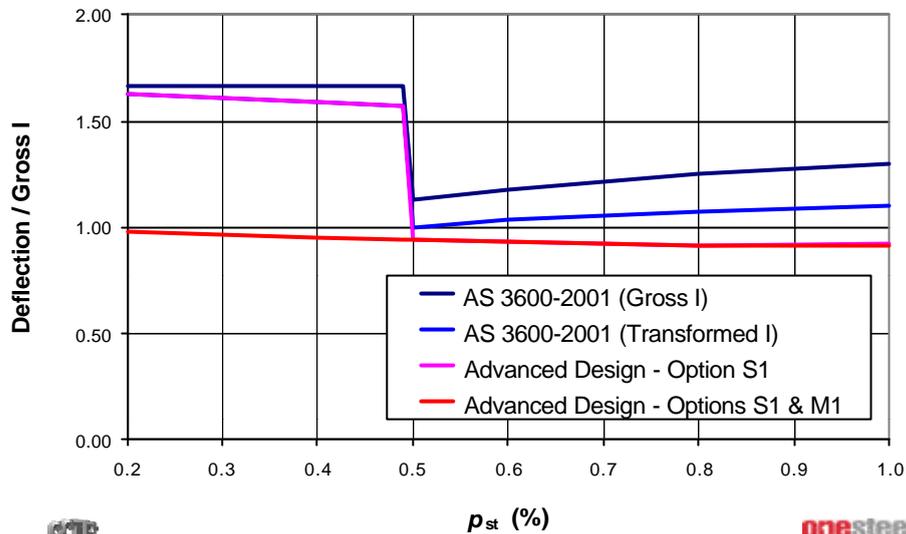
Simply-supported slab,  $Q = 3 \text{ kPa}$ ,  $\rho_{sc} = \rho_{st}$



In this graph, the deflections have been divided by the corresponding deflection determined by simplified calculation using AS 3600-2001 (Gross I), which is shown as the top line in the previous slide. It shows that for  $\rho_{st} > 0.005$ , reductions of at least 35% can be expected using the advanced design options S1 and M1 compared with the normal way simplified calculations are performed in accordance with AS 3600-2001.

## Comparisons

**Continuous slab,  $Q = 3 \text{ kPa}$ ,  $p_{sc} = p_{st}$**

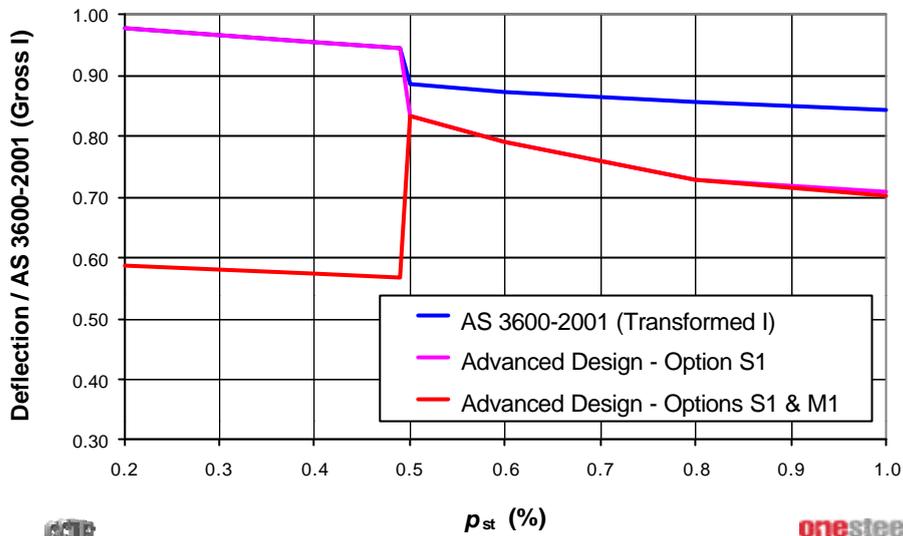


The discontinuity at 0.5% reinforcement ratio is due to the new upper limit placed on the effective second moment of area,  $I_{c,max}$ , of  $0.6I$  for  $p_{st} < 0.005$  in accordance with Clause 8.5.3.1 of AS 3600-2001.

Since for option M1 the degree of cracking at each cross-section must be evaluated taking into account shrinkage-induced tensile stress in the concrete, it is proposed in this graph that it is not necessary to apply the  $0.6I$  restriction. (This issue is currently under investigation.) In any case, results are shown for  $p_{st}$  down to 0.002 which is enough to satisfy the minimum bending strength requirement in Clause 8.1.4.1 of AS 3600-2001. However, it may not be enough for flexural crack control (Clause 9.4.1) or crack control for shrinkage and temperature effects (Clause 9.4.3), and the need for the discontinuity becomes even less important.

## Comparisons

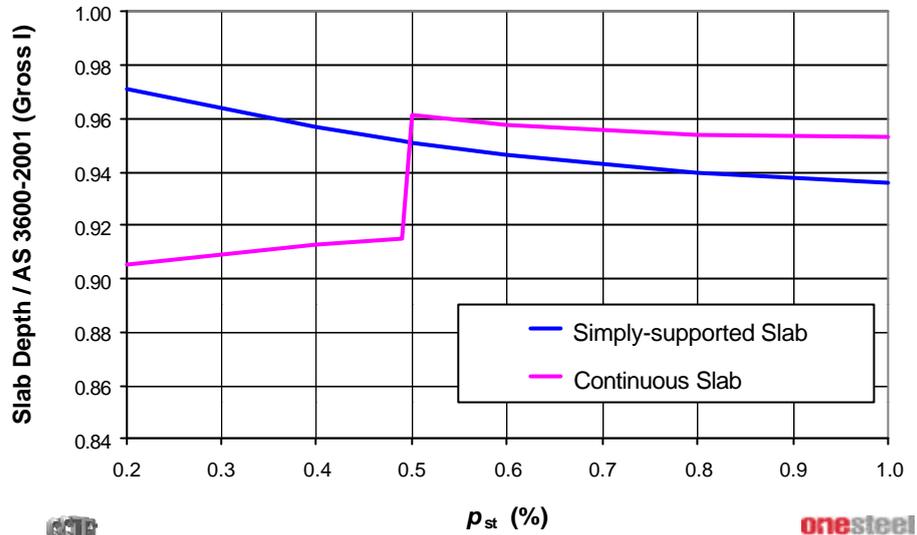
Continuous slab,  $Q = 3 \text{ kPa}$ ,  $p_{sc} = p_{st}$



In this graph, once again the deflections have been divided by the corresponding deflection determined by simplified calculation using AS 3600-2001 (Gross I), which is shown as the top line in the previous slide. It shows that for  $p_{st} > 0.005$ , reductions of about 20-30% can be expected by using options S1 and M1.

## Comparisons

### Slab depth using Advanced Design™ – Options S1 & M1



This graph has been produced to show the significant reduction in overall slab depth that might be achieved by using advanced design options S1 and M1. It can be seen that for  $p_{st} > 0.005$ , reductions of about 5% can be expected. It can be shown that alternatively the span of the slabs studied could be increased by 5%.

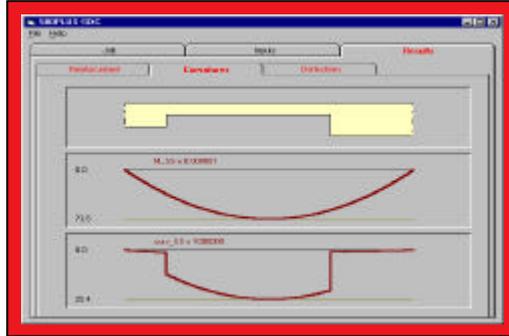
## Conclusions

- **Advanced Design™**
  - Advanced Design™ options can predict up to 30% smaller one-way slab deflections than simplified calculations to AS 3600–2001
- **Implications**
  - 5% thinner or longer-spanning reinforced-concrete slabs
  - Effective utilization of 500 MPa reinforcing steels
  - Economic viability of reinforced-concrete slabs significantly improved
- **Future**
  - Slabs with two-way action will also benefit



# Advanced Design™ for Slab Deflections

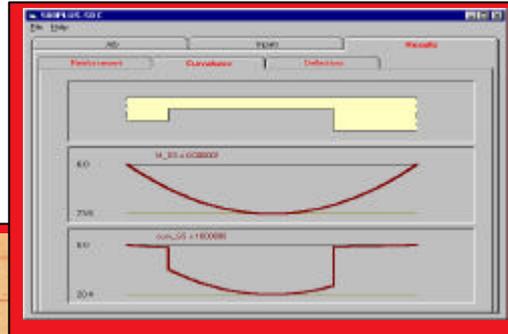
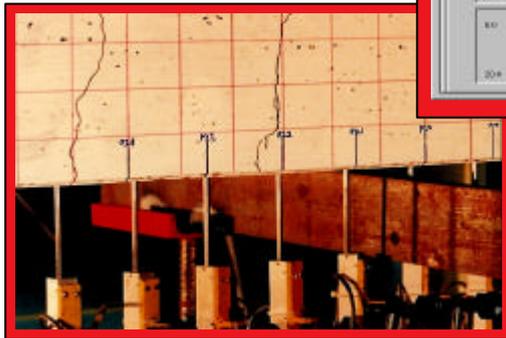
## Software Demonstration 500PLUS-SDC™



A trial version of the new software is included elsewhere on this CD.

# Advanced Design™ for Slab Deflections

The



End

