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Design Booklet RCB-1.1(1)

Crack Control of Beams

Part 1: AS 3600 Design

OneSteel Reinforcing

Guide to Reinforced Concrete Design

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Preface

This design booklet is a part of OneSteel Reinforcing's Guide to Reinforced Concrete Design that has been produced to promote the superiority of OneSteel Reinforcing's reinforcing steels, products and technical support services. The Guide covers important issues concerning the design and detailing of Reinforced Concrete Buildings, Residential Slabs and Footings, and Concrete Pavements. The use of 500PLUS® reinforcing steels is featured in the booklets. Special attention is given to showing how to get the most benefit from these new, superior high-strength steels.

The design booklets of the Reinforced Concrete Buildings Series have each been written to form two separate parts: Part 1- AS 3600 Design which provides insight into major new developments in AS 3600; and Part 2 – Advanced Design™ Using 500PLUS® which leads to significant economic advantages for specifiers of OneSteel reinforcing steel. These booklets are supported by 500PLUS computer software that will prove to be indispensable to design engineers who use AS 3600.

To control flexural cracking, the Concrete Structures Standard AS 3600-1994 required only the maximum spacing and concrete cover of the tension reinforcement to be limited, and often this did not guarantee acceptably narrow cracks. The new edition of the Concrete Structures Standard AS 3600-2000[1] will allow 500 MPa reinforcing steels to be used in design. This will inevitably lead to higher steel stresses under serviceability conditions, thereby increasing the importance of designing for crack control. New design provisions, for crack control of beams in a state of either flexure or tension, that are proposed for inclusion in AS 3600-2000 are reviewed in this design booklet. They have essentially come from Eurocode 2, and their use needs to be well understood by designers in order to allow the full benefit of the increase in steel yield strength to be gained, leading to a significant reduction in steel area. This may require judicious detailing of the bars, crack control improving with a reduction in either bar diameter or bar spacing. A computer program 500PLUS-BCC™ (BCC standing for Beam Crack Control) is being released with this design booklet, and is the first from the 500PLUS software suite. Section 6 of this booklet is effectively the User Guide for the program. Further research is proceeding that will allow these design provisions to be improved upon when using 500PLUS Rebar, and more advanced rules will be found in Part 2[2] of this design booklet.

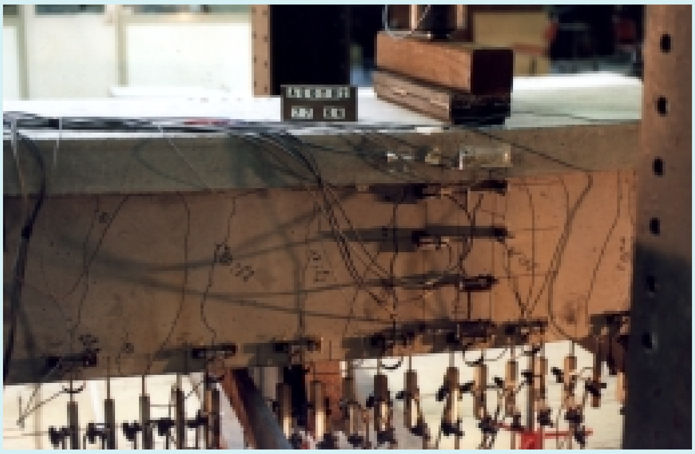
[1] AS 3600-2000 is expected to be published this year by Standards Australia.
[2] In course of preparation.

1. SCOPE AND GENERAL

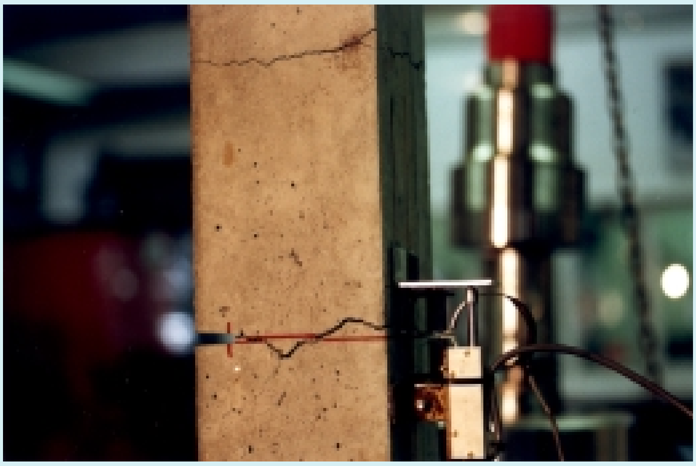
1.1 Scope

This design booklet is concerned with the control of flexural and tensile cracking in reinforced-concrete beams designed in accordance with AS 3600-2000 (see Appendix A, Referenced Australian Standards). It provides rules essential for being able to efficiently detail the main (longitudinal) reinforcement. The need for these rules is largely the result of including 500 MPa as a standard strength grade in AS 3600-2000. This is a significant increase over the other grades of 400 MPa for bars and 450 MPa for welded mesh. Rules for using bundled bars are included, but bundling increases the effective diameter of the bars and this can make crack control more difficult.

The reinforcement must consist of deformed bars with a rib geometry that satisfies AS 1302 or AS 1303 for 400 MPa and 450 MPa steel, respectively, or it must be 500PLUS® Rebar. The bars may be used separately or form a welded mesh depending on their availability. The standard strength grade may be either 400, 450 or 500 MPa.



(a) T-beam in flexure



(b) Truss chord in tension

Figure 1.1 Cracking in Reinforced-Concrete Beams Due to Direct Loading

Rules for controlling cracking due to direct loading arising from self-weight of component elements and externally applied loads (see Fig. 1.1), imposed deformation (e.g. settlement of supports) or restrained deformation (e.g. concrete shrinkage) are covered. Controlling cracking under imposed or restrained deformation may require the use of articulation joints, particularly if the location of stiffening elements (walls and cores) is undesirable in this regard. The use of these joints is beyond the scope of this booklet, but guidance is available in the literature [1,22]. The need for any special provisions for controlling cracking at locations of mechanical couplers is also an issue that is not covered.

In situations when bending is the main action effect, flexural cracks will form. These cracks appear in the tension face, and extend more or less vertically in a small region over interior supports and in mid-span regions of continuous beams. Flexure-shear cracks form in regions adjacent to the flexural cracks where the shear force is more significant. The flexure-shear cracks form from short vertical flexural cracks, but become inclined in the beam web. The design rules contained herein for flexural elements are intended to control the width of both of these types of cracks. Cracking usually near the level of the elastic neutral axis, due to diagonal tension caused when shear force is the dominant action effect, is not addressed, although some guidance on this subject is given in Eurocode 2 [2].

Special issues relating to the calculation of steel stresses in non-flexural members including deep beams, corbels, etc. preclude this booklet being used directly to design these elements for crack control, although some of the information contained herein is relevant.

Crack control at openings through beam webs or flanges is not specifically covered, although some advice on the subject is given elsewhere [23]. Nor are the possible effects of internal voids in the slabs forming the beam flanges (e.g. for the passage of building services or to reduce weight), or discontinuities such as at recesses and local changes in depth considered. In such situations, the reinforcement should in general exceed the minimum requirements specified herein.

1.2 General

The design rules presented herein are based on new rules in Eurocode 2 for crack control. The normal strength grade for reinforcement in Eurocode 2 is 500 MPa, which will be permitted in AS 3600-2000, and Eurocode 2 is currently the most appropriate design document to form a basis on which to develop Australian rules [3,4]. However, it will be pointed out herein that important issues concerning use of some of the rules in Eurocode 2 have not been made clear. Therefore, these aspects are still open to interpretation, and it has not been possible to simply transpose the European rules into this design booklet. More development work is still needed.

The rules in Eurocode 2 also address the design of prestressed beams, but no changes have yet been proposed to the rules for these elements in AS 3600-2000.

Eurocode 2 allows a tiered approach to design: (i) crack width formulae can be used to keep crack widths below the design crack width (normally 0.3 mm – see Section 3.2); or (ii) simplified rules derived directly from the crack width formulae provide acceptable values of bar diameter and bar spacing depending on the maximum stress in the steel under service loads. Both of these design approaches or methods are discussed, although only the simplified rules (partly modified) have been proposed for AS 3600-2000. The objective behind explaining the crack width formulae is to provide engineers with the opportunity to understand the background to the simplified method. This reduces the likelihood of the simplified rules being followed like a recipe, and helps designers appreciate somewhat the significance of major design parameters such as bar diameter.

Cracking of concrete is a major topic with numerous complex facets. The reader is referred to other documents for a broader insight into the topic, e.g. [5,6,7]. Some documents, which explain some of the development work behind the design rules in Eurocode 2, are also worth citing [8,9,10,11].

Cracking can be caused by any of a variety of actions, which include loads applied during either construction or the in-service condition, foundation movements, temperature changes and gradients, shrinkage and creep of concrete, etc. The calculation of design action effects for crack control design can be a complex exercise in its own right for any of these situations. Bending moments and/or axial forces need to be calculated at critical sections. It is beyond the scope of this document to address this topic in any detail, and the reader needs to refer to specialist literature and computer software manuals for guidance. Examples that cover the calculation of design action effects for beams subjected to thermal gradients and restrained shrinkage, which should be of particular interest to designers since these are often “unaccounted for effects” that cause unexpected cracking problems, can be found in references [8,11,12,21].

2. TERMINOLOGY

Some important terminology used in this booklet is summarised in this Section.

Action

An agent including direct loading, imposed deformation or restrained deformation, which may act on a structure or its component elements.

Action effects

The forces and moments that are produced in a structure or in its component elements by an action.

Cracked section

A section of a reinforced-concrete beam cracked over part or all of its cross-sectional area, and where the tensile strength of the concrete is ignored in design.

Critical steel content

Minimum amount of steel required in the tensile zone of a tensile or flexural element for multiple cracks to form in a uniform stress zone.

Direct loading

Loading on a structure that includes the self-weight of its component elements and externally applied loads.

Fully cracked section

A section of a reinforced-concrete beam cracked over all of its cross-sectional area, and where the tensile strength of the concrete is ignored in design.

Imposed deformation

Deformation imposed on a beam by its supports.

Main reinforcement

Reinforcement provided by calculation to resist action effects, irrespective of its direction.

Non-flexural elements

Deep beams, footings, pile caps, corbels, etc. as defined in Clause 12.1.1.1 of AS 3600-1994.

Restrained deformation

Deformation of a beam resulting from concrete shrinkage or temperature variations restrained by its supports (or by embedded reinforcement).

Stabilised crack pattern

The final crack pattern that forms in a reinforced-concrete element.

State of flexure

The condition when the tensile stress distribution within the section prior to cracking is triangular, with some part of the section in compression.

State of tension

The condition when the whole of the section is subject to tensile stress.

Uncracked section

A section of a reinforced-concrete beam uncracked over its entire cross-sectional area, and where the tensile strength of the concrete is included in design.

3. DESIGN CONCEPTS & MODELS

3.1 General

Cracking of concrete will occur whenever the tensile strength of the concrete is exceeded. This is inevitable in normal reinforced-concrete structures, and once formed the cracks will be present for the remainder of a structure's life. Because cracks affect the serviceability of a building, the limit state of excessive crack width needs to be considered in design.

This booklet is concerned with the design of reinforced-concrete beams for crack control after the concrete has hardened. This essentially involves ensuring firstly that the cracks form in a well-distributed pattern, normally very early in the life of a structure, and secondly that they do not become excessively wide while a building is in service. Moreover, this first condition requires that the tension reinforcement must have a minimum tensile capacity when the cracks form. The second condition requires that the tensile stresses in the reinforcement do not become too large at any time in the life of a structure.

Cracking can occur when the beams are directly loaded. This may be immediately after the temporary props are removed during construction and the beams must support their self-weight for the first time. Imposed or restrained deformations can also cause cracking. Support settlement is an example of an imposed deformation that tends to cause flexure in a beam. Shrinkage of concrete or temperature changes can cause the occurrence of restrained deformation. These actions can cause significant flexural or direct tensile stresses to develop in the hardened concrete. Without steel reinforcement, a cracked section cannot provide flexural or tensile restraint to the adjoining concrete segments, and crack control is impossible. Reinforcing steel is required in beams to control cracking under these circumstances. The way in which tension reinforcement can control cracking in a beam subjected to restrained deformation arising from concrete shrinkage is broadly illustrated in Fig. 3.1.

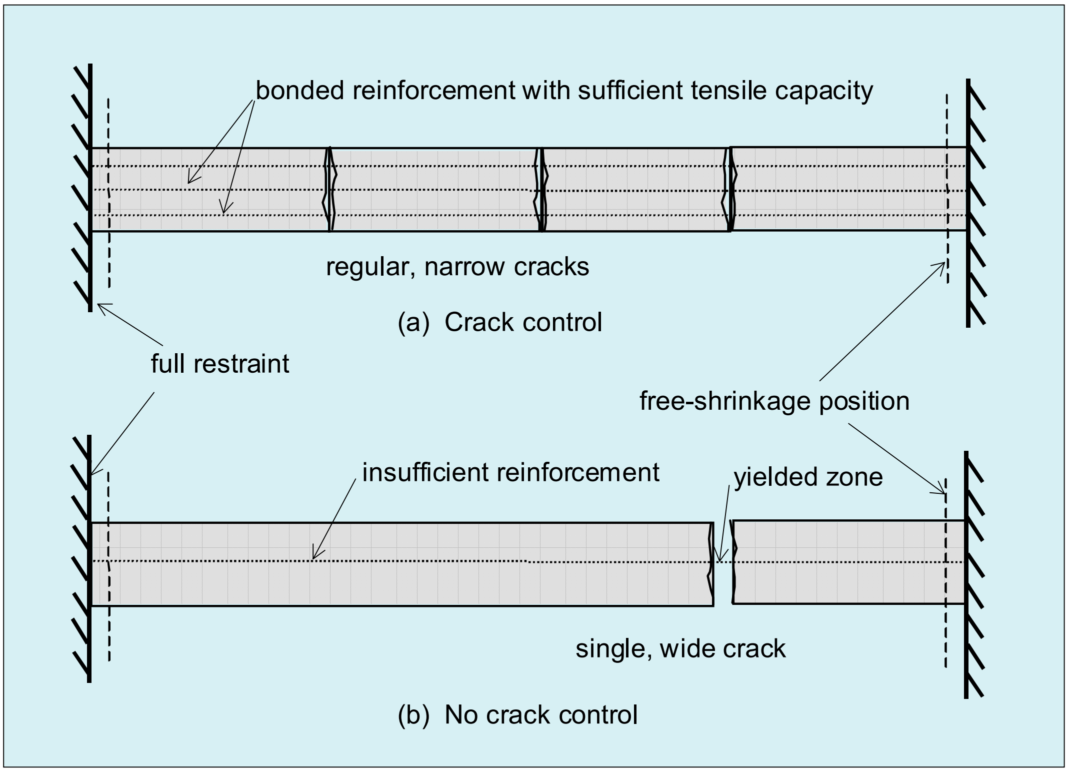


Figure 3.1 Control of Cracking Caused by Restrained Deformation

Prior to the concrete hardening, plastic cracking can arise from either plastic settlement (which results from differences in density of the mix components), or plastic shrinkage (which results from evaporation of bleed water), or a combination of both.

Plastic settlement cracking results from uneven settlement of the concrete mix, which can occur at changes in thickness of the concrete, and near discrete obstructions such as steel reinforcing bars or larger pieces of aggregate. There are many established practices for reducing plastic settlement. These include using cohesive concrete mixes, using air-entrainer, increasing cover to top reinforcement, and adjusting the placing rate.

Plastic shrinkage cracking is likely to occur when there is rapid loss of bleed water, which might occur by evaporation or possibly by absorption into the adjacent formwork or subgrade. Experienced practitioners will normally take measures to avoid these problems, including sealing the forms, erecting windbreaks and covering the exposed surface of the concrete as early as possible after finishing.

3.2 Crack Width Limits

As a rule, a design engineer should aim to detail beams such that tensile strains are distributed over a large number of narrow cracks rather than a small number of wide cracks in the surface of the concrete. The control of surface cracking is particularly important in certain situations. The most common of these is where the surface will be visible, as excessive crack widths can give an overall impression of poor quality and can limit the types of floor coverings that can be successfully used. Crack control is also important for durability where the cracks would provide pathways for the ingress of corrosive substances such as water.

Where there are no special requirements such as watertightness, Eurocode 2 recommends that a limit of 0.3 mm under quasi-permanent (long-term) loads should normally be satisfied. This design crack width, w_k, is intended to be a characteristic value with only a 5% probability of being exceeded (see Section 3.5.2). It is recognised that for reinforced-concrete beams in a dry environment corresponding to the interior of buildings for normal habitation or offices, durability is not normally a concern that affects the choice of design crack width.

In areas where the top surface of the concrete is exposed to foot or vehicular traffic, such as in a carpark or a footbridge, the edges of cracks can become damaged and frayed, and appear considerably wider at the surface. In this type of situation, it is prudent to control crack widths even more tightly.

3.3 Cracking of Reinforced-Concrete Elements

3.3.1 Introduction

The development of cracking in reinforced-concrete elements by tension or flexure is a complex topic. A brief, general account is given here of this behaviour before specifically discussing cracking of reinforced-concrete beams in Section 3.4.

Cracking is considered to have occurred under conditions of direct tension, referred to herein more simply just as “tension”, if tensile stresses existed across the entire cross-section immediately prior to cracking. According to this definition, bending moment can be carried by a section deemed to be in tension, provided there is no compressive stress. Stresses induced by differential temperature or shrinkage that develop through the depth of a reinforced-concrete element can complicate the resultant stress distributions prior to cracking. Their existence increases the difficulty of predicting the onset of cracking, and they will be ignored here.

Many different actions can cause tension or flexure to develop in a section of a reinforced-concrete element, e.g. applied loads, support settlement, concrete shrinkage, or thermal expansion or contraction. The support or restraint that is provided at the boundaries of the element can also significantly affect the stress distribution at each of its cross-sections. However, in order to simplify this discussion, it will generally be assumed that the elements support applied loads of known magnitude. It will also be generally assumed that they are statically determinate, whereby the resultant tensile force and/or bending moment acting at every cross-section is known.

3.3.2 Cracking in Tension Elements

3.3.2.1 Classical Theory

It is instructive to consider the behaviour of a reinforced-concrete tension element with a longitudinal reinforcing bar placed concentrically in its cross-section and loaded at each end by a known force (see Fig. 3.2). The classical theory, which describes the development of cracking in such an element, is well accepted, and has often been used as a basis for deriving equations for predicting crack widths [11,13]. The type of approach taken is described as follows.

When the bar is loaded in tension, some bond breakdown occurs between the bar and the concrete near the ends of the element. Further in, a uniform strain distribution is assumed to develop, and slip between the steel and concrete remains zero.

The first crack forms at the weakest section somewhere in the region of uniform strain when the tensile strength of the concrete is reached. This assumes that the tensile capacity of the bar exceeds that of the concrete, noting that when they are equal this is referred to as the critical steel content. Otherwise, the bar will fail in tension outside the concrete before the concrete can crack.

Just like at the ends of the concrete element, the force in the steel bar at the crack equals the applied load, while the concrete is unstressed at the crack faces. Also, slip occurs and bond stress, τ, develops between the concrete and the steel bar over a transfer length, l_{tr}, each side of the crack. It is by bond that stress is transferred into the concrete. Depending on the overall length of the element in relation to the transfer length, other cracks can form at slightly higher loads.

Theoretically, the spacing between cracks that form adjacent to each other cannot be less than l_{tr}, and nor can it exceed $2l_{tr}$. This is explained as follows. Consider the two cracks that have formed at cross-sections 1 and 2 in Fig. 3.2. A new crack can only form between them if they are at least $2l_{tr}$ apart. If the spacing is just above this limit and another crack forms, then the crack spacing will be close to l_{tr}. Thus, it can be written that:

$$s_{cr.min} = l_{tr} \qquad 3.3.2.1(1)$$

$$s_{cr.max} = 2l_{tr} \qquad 3.3.2.1(2)$$

It follows from equilibrium of longitudinal forces that if τ_m is the average bond stress over the transfer length l_{tr}, f_t is the tensile strength of concrete and Σ_o is the bar perimeter, then:

$$l_{tr} = \frac{A_c f_t}{\tau_m \Sigma_o} \qquad 3.3.2.1(3)$$

Substituting bar perimeter $\Sigma_o=4A_{st}/d_b$ and reinforcement ratio for tension $p_s=A_{st}/A_c$, it follows that:

$$l_{tr} = \frac{d_b f_t}{4\tau_m p_s} \qquad 3.3.2.1(4)$$

and from Eqs 3.3.2.1(2) and 3.3.2.1(4), the maximum crack spacing becomes:

$$s_{cr.max} = \frac{d_b f_t}{2\tau_m p_s} \qquad 3.3.2.1(5)$$

Finally, crack width equals the elongation of the steel between two adjacent cracks less the elongation of the concrete, and one can write:

$$w_{max} = s_{cr.max}(\varepsilon_{sm} - \varepsilon_{cm}) \qquad 3.3.2.1(6)$$

where ε_{sm} and ε_{cm} are the mean steel and concrete strains over transition length l_{tr}.

At the end of the transition length, the steel bar is fully bonded to the concrete, and the tensile force in the steel at this location, T_b', is given by:

$$T_b' = T_b \frac{np_s}{1+np_s} \qquad 3.3.2.1(7)$$

where T_b is the tensile force in the bar at cracked sections. Assuming a uniform bond stress over transition length l_{tr}, it follows that the average strain in the steel bar, ε_{sm}, is given by:

$$\varepsilon_{sm} = \frac{1}{2E_s}\left(\frac{T_b}{A_{st}} + \frac{T_b'}{A_{st}}\right)$$

and from Eq. 3.3.2.1(7), it follows that,

$$\varepsilon_{sm} = \frac{f_s}{2E_s}\left(\frac{1+2np_s}{1+np_s}\right) \qquad 3.3.2.1(8)$$

where f_s=T_b/A_{st}.

From Eq. 3.3.2.1(8) it can be seen that the average steel strain, ε_{sm}, is a function of the reinforcement ratio p_s=A_{st}/A_c. It follows that ε_{sm} increases with p_s, and in practice, the term in brackets in Eq. 3.3.2.1(8) might reach a maximum value of about 1.3. Therefore, a reasonable upper estimate for ε_{sm} is 0.65f_s/E_s. It follows that, if elongation of the concrete is ignored, i.e. ε_{cm}=0, then an approximate formula for maximum crack width, w_{max}, can be written as follows using Eqs 3.3.2.1(6) and 3.3.2.1(8):

$$w_{max} = s_{cr.max}\frac{0.65 f_s}{E_s} \qquad 3.3.2.1(9)$$

and then substituting Eq. 3.3.2.1(5) into Eq. 3.3.2.1(9) to give:

$$w_{max} = \frac{d_b f_t}{2\tau_m p_s}\frac{0.65 f_s}{E_s} \qquad 3.3.2.1(10)$$

This important relationship shows that, all other parameters remaining the same, if the bar diameter d_b is increased, then the steel stress f_s at a cracked section must proportionally decrease if the maximum crack width is to remain unchanged. Therefore, the interdependence between crack width, bar diameter and steel stress has been approximately established using classical theory for a reinforced-concrete tensile element with a stabilised crack pattern.

Equation 3.3.2.1(10) for maximum crack width can be refined by including terms to account for the concrete strain. At an uncracked section, it can be written that:

$$f_c = \frac{f_s p_s}{(1+np_s)} \qquad 3.3.2.1(11)$$

where f_c is the tensile stress in the concrete, which has a maximum value of f_t, whereby it follows from Eq. 3.3.2.1(11) that:

$$f_s = \frac{f_t(1+np_s)}{p_s} \qquad 3.3.2.1(12)$$

It will again be assumed that the bond stress τ is uniform or constant over the transition length. Then it can be written:

$$\varepsilon_{cm} = \frac{1}{2}\varepsilon_c = \frac{1}{2}\varepsilon_s = \frac{1}{2}\frac{f_s}{E_s} \qquad 3.3.2.1(13)$$

which leads, from Eq. 3.3.2.1(12), to:

$$\varepsilon_{cm} = \frac{1}{2}\frac{f_t(1+np_s)}{p_s E_s} \qquad 3.3.2.1(14)$$

As a further refinement, the effect of concrete shrinkage can be included to give:

$$\varepsilon_{cm} = \frac{1}{2}\frac{f_t(1+np_s)}{p_s E_s} - \frac{1}{2}\frac{\varepsilon_{cs}}{(1+np_s)} \qquad 3.3.2.1(15)$$

where ε_{cm} is the free shrinkage strain of the concrete and will be a negative value. It follows from Eqs 3.3.2.1(6) and 3.3.2.1(15) that Eq. 3.3.2.1(10) can be written more generally as:

$$w_{max} = \frac{d_b f_t}{2\tau_m p_s}\left(\frac{0.65 f_s}{E_s} - \frac{1}{2}\frac{f_t(1+np_s)}{p_s E_s} - \frac{1}{2}\frac{\varepsilon_{cs}}{(1+np_s)}\right) \qquad 3.3.2.1(16)$$

Returning to the case of a single crack, the equations necessary to calculate its width, w, will be formulated assuming constant bond stress. Firstly, similar to Eq. 3.3.2.1(6) it can be written that:

$$w = 2l_{tr}(\varepsilon_{sm} - \varepsilon_{cm}) \qquad 3.3.2.1(17)$$

It follows from Eqs 3.3.2.1(4) and 3.3.2.1(12) that Eq. 3.3.2.1(17) becomes:

$$w = \frac{d_b f_s}{2\tau_m}\frac{1}{(1+np_s)}(\varepsilon_{sm} - \varepsilon_{cm}) \qquad 3.3.2.1(18)$$

The average steel strain, ε_{sm}, is given by Eq. 3.3.2.1(8), while the average concrete strain, ε_{cm}, can be shown to equal:

$$\varepsilon_{cm} = \frac{f_s}{2E_s}\frac{np_s}{(1+np_s)} \qquad 3.3.2.1(19)$$

Substituting Eqs 3.3.2.1(8) and 3.3.2.1(19) into Eq. 3.3.2.1(18) gives:

$$w = \frac{d_b f_s}{2\tau_m}\frac{1}{(1+np_s)}\left(\frac{f_s}{2E_s}\right) \qquad 3.3.2.1(20)$$

Like for Eq. 3.3.2.1(16), the effect of concrete shrinkage can be included to give:

$$w = \frac{d_b f_s}{2\tau_m}\frac{1}{(1+np_s)}\left[\frac{f_s}{2E_s} - \frac{1}{2}\frac{\varepsilon_{cs}}{(1+np_s)}\right] \qquad 3.3.2.1(21)$$

Equations 3.3.2.1(16) and 3.3.2.1(21) can be further generalised by incorporating the effects of a non-linear bond stress relationship based on test data. However, it is beyond the scope of this booklet to consider the derivation of more accurate forms of these equations. It can be shown that by simply substituting w=0.3mm, τ_m=5.5 MPa (a representative value for deformed bars, although dependent on factors such as the tensile strength of the concrete, confinement, etc. [14]) and $\varepsilon_{cs}/(1+np_s)$=-250 $\mu\varepsilon$ into Eq. 3.3.2.1(21) leads to a very similar relationship between d_b and f_s to that given in Eurocode 2, at least for bar diameters up to 20 mm. The relationship in Eurocode 2 is for simplified crack control design of either tension or flexural elements.

3.3.2.2 Minimum Reinforcement

Ignoring shrinkage restraint of the reinforcing steel, the theoretical static load at first cracking, T_{cr}, is given by [15]:

$$T_{cr} = f_t(A_c + nA_{st}) \qquad 3.3.2.2(1)$$

For the situation shown in Fig. 3.2, where the tensile force T_b is shown being applied to both ends of the steel bar protruding from the concrete, a crack will not form unless $T_b \geq T_{cr}$. Thus, it can be written, assuming for simplicity that the reinforcing steel behaves elastic-plastically with a yield strength of f_{sy}:

$$f_{sy}A_{st} > f_t(A_c + nA_{st}) \qquad 3.3.2.2(2)$$

and transforming gives:

$$A_{st} > \frac{f_t A_c}{f_{sy} - nf_t} \qquad 3.3.2.2(3)$$

With this condition satisfied, at least one crack can form in the concrete. Whilst this equation is only approximate because nominal material properties are implied, the real intention of deriving this equation is to show when multiple cracks will form. Multiple cracks are desirable since on average they are finer the more cracks that form.

Nevertheless, from a design perspective, it is theoretically not necessary to comply with Eq. 3.3.2.2(3), if at the strength limit state the design tensile force, T^*, is less than T_{cr}. This is because then cracking would theoretically be avoided altogether (ignoring shrinkage, etc.).

The derivation of an equation for calculating the minimum reinforcement of a redundant tensile element like that shown in Fig. 3.1 is quite different to above. Shrinkage restraint of the concrete must be considered, noting that prior to cracking the steel stress is zero. However, if satisfactory crack control is to be achieved Eq. 3.3.2.2(2) must still normally be satisfied, since only then can multiple cracks form. The intention is to avoid yielding of the reinforcement, because crack widths cannot be controlled if this occurs.

As a final comment, if the simple equations given above are used for design, then conservative estimates for the concrete tensile and steel yield strengths need to be used.

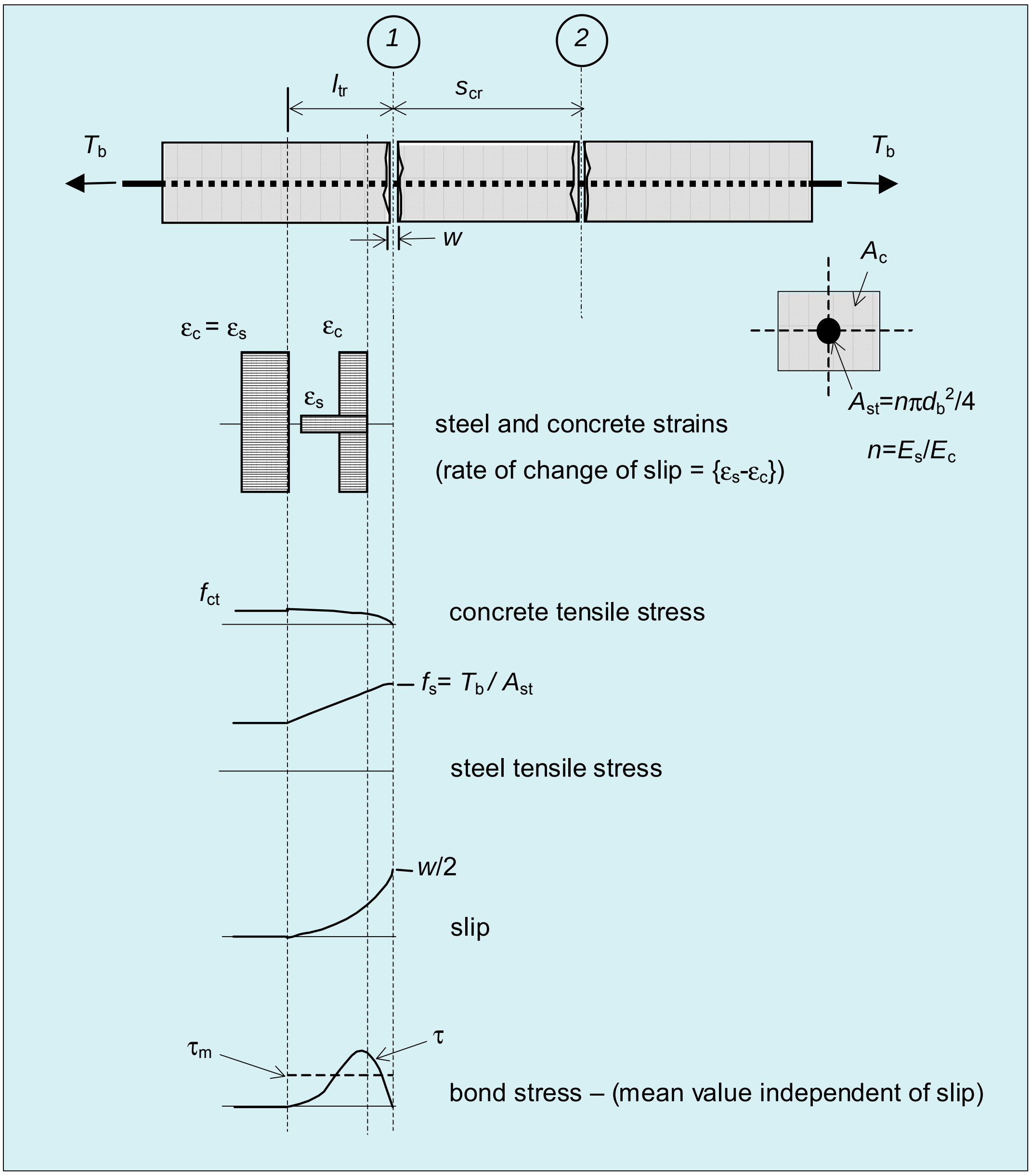


Figure 3.2 Cracking in Tension [15]

3.3.3 Cracking in Flexural Elements

3.3.3.1 General

The classical theory used in Section 3.3.2 to describe the development of cracking and the calculation of crack widths in tension elements is based on a large number of simplifying assumptions. For example, the tensile stress in the concrete would not be uniform, but would vary significantly across the width and depth of the element at cross-sections away from the cracks. This is due to the local nature of the bond transfer mechanism, which involves part of the force in the steel at cracks dispersing into the surrounding concrete creating a sort of stress bulb in the concrete. The area of concrete that can be assumed effective in tension at critical sections would clearly depend on this effect. It is also assumed that cracks have constant width with parallel sides through the width and depth of the element.

For flexural elements, which can be under a combination of bending and tension, the complexity of the internal stress distributions prior to and after cracking is further increased compared with tension elements. This makes the definition of an effective area of concrete in tension an even greater issue. Nor can crack width be assumed to be constant over the depth of the element, naturally equalling zero in zones of compressive stress for example. Moreover, tests have shown that concrete cover can have a significant affect on surface crack widths. Attempts to apply equations of a similar form to Eqs 3.3.2(16) and 3.3.2(21) to predict crack widths in beams have generally demonstrated a need to reduce the effect of d_b and p_s. This has led to the development of simpler empirical equations. A statistical approach has been used because of the large variability between test results and predicted values.

It is beyond the scope of this document to review the background to the different crack width equations that have been developed. Identifying the main variables is more contentious than for tension elements. However, the crack width equations proposed in Eurocode 2 for flexural elements are based on a modified form of Eq. 3.3.2.1(6). This has been made it possible to design either tension or flexural elements using the same basic design equations, with different values for some of the coefficients. These equations are presented in Section 3.5.

3.3.3.2 Minimum Reinforcement

It was stated in Section 3.3.2.2 that one basic principle that must be complied with to control cracks in tensile elements is to avoid yielding the reinforcement. This same principle applies for flexural elements. The most elementary way of expressing this requirement is to write:

$$M_{sy} > M_{cr} \qquad 3.3.3.2(1)$$

where M_{sy} is the moment capacity of the flexural element at a cracked section with the reinforcing steel at its yield strain, and M_{cr} is the cracking moment. Here it is conservative to ignore the effects of shrinkage, because this can reduce the cracking moment.

However, multiple cracks will not form under moment gradient conditions if Eq. 3.3.3.2(1) is only just satisfied at the peak moment cross-section of the critical positive or negative moment region. Therefore, a more stringent requirement may actually be required in practice.

Expanding on Eq. 3.3.3.2(1), one can write:

$$A_{st} f_{sy} \chi d > \frac{\xi f_t b D^2}{6} \qquad 3.3.3.2(2)$$

where ξ ($\geq$1) is a parameter to account for moment gradient effects, and χd is the lever arm of the internal force couple (noting that for lightly-reinforced elements $\chi \approx 0.9$). Transforming Eq. 3.3.3.2(2) gives:

$$\frac{A_{st}}{bd} > \frac{\xi f_t}{6 \chi f_{sy}} \left(\frac{D}{d} \right)^2 \qquad 3.3.3.2(3)$$

It is interesting to note that Eq. 3.3.3.2(3) shows that the minimum value of the reinforcement ratio,

$p=A_{st}/bd$, required for crack control of a section in bending, is independent of the overall depth, D, depending instead on D/d.

Tests shows that shallow, reinforced-concrete rectangular elements (beams and slabs) with a low value of reinforcement ratio, A_{st}/bd, attain significantly greater bending strengths than Eq. 3.3.3.2(3) would indicate [16]. Typically, their overall depth, D, could be up to 200 mm or more for this to still hold. This is because under static loading the tensile strength of the concrete makes a significant contribution towards the moment capacity of the critical section compared with the reinforcement. Therefore, crack growth tends to be stable in shallow, lightly-reinforced flexural elements.

On the other hand, the moment capacity of deep, lightly-reinforced flexural elements is effectively controlled by the tension reinforcement. Deep elements (e.g. D = 800 mm) with a reinforcement ratio satisfying Eq. 3.3.3.2(3) can actually fail suddenly without the tensile capacity of the reinforcement being developed. The failure mechanism involves the first flexural crack penetrating the compressive stress zone with very little rotation at the critical section, which does not remain plane. By placing some light horizontal reinforcement in the vertical sides of the element, i.e. skin reinforcement, the flexural behaviour of deep elements can be substantially improved. It is standard to require skin reinforcement for crack control detailing (see Fig. 5.1), although the fact that it contributes towards avoiding sudden collapse by preventing unstable crack propagation is not well known. Ozbolt and Bruckner [16] imply that deep beams with skin reinforcement can be satisfactorily designed using Eq. 3.3.3.2(3).

3.4 Actions Causing Cracking of Reinforced-Concrete Beams

Some examples are given here to describe the way cracking can develop in reinforced-concrete beams. Cracking can be caused by any of a number of actions, which include loads applied during either construction or the in-service condition, foundation movements, temperature changes and gradients, shrinkage and creep of concrete, etc.

A beam may be deemed to be in a state of either tension or flexure prior to the onset of cracking (see Fig. 3.3). This state determines the way in which the beam should be designed for crack control, as will be described in Section 3.5. Recapping from Section 3.3.1, the internal forces in a beam prior to cracking can be a combination of tensile force and bending moment. A state of tension is assumed to exist if there is no compressive stress at the section of concern (see Fig. 3.3(b)).

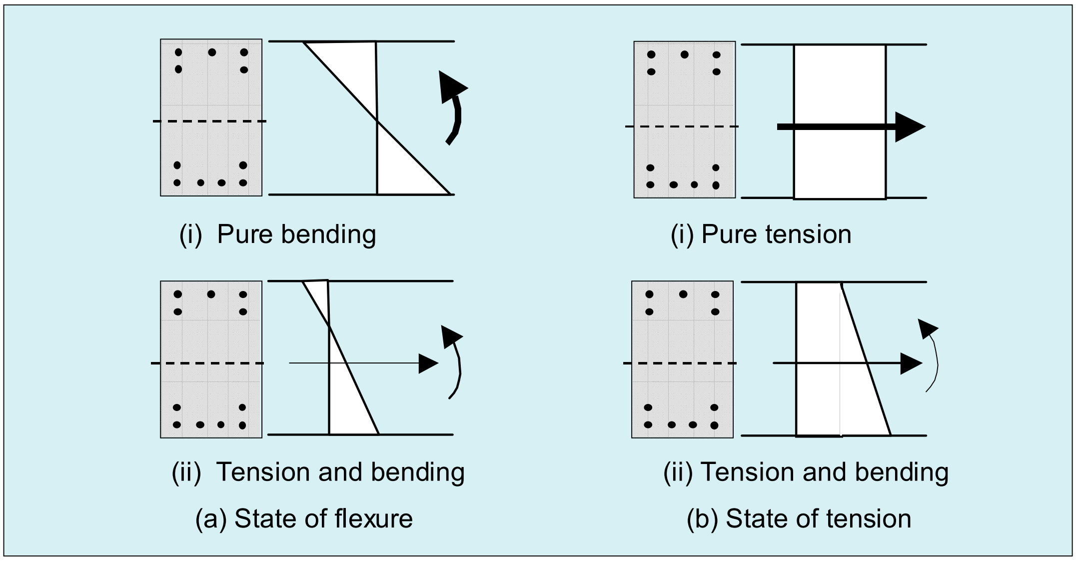


Figure 3.3 State of Flexure or Tension Prior to Cracking

The types of actions that commonly cause a state of tension or flexure in a beam are shown in Fig. 3.4. These are briefly explained as follows.

Direct loading is a common cause of a state of flexure in a reinforced-concrete beam with a normal span-to-depth ratio (see Fig. 3.4(a)). Many engineers probably regard this as the most important situation for designing for crack control. However, other actions can cause significant cracking in relatively lightly-loaded structures if their effect is overlooked during design.

Support or foundation movement is the type of action that is often overlooked by design engineers, even though this can lead to significant serviceability problems if the structure is not appropriately designed. Articulation joints may be required if the movements anticipated are very large. Settlement of a beam support is an example of an *imposed deformation*. It can arise due to differential soil settlement, in which case the deformation is externally imposed on the structure. Differential column shortening is an example of an internally imposed deformation. This can have the same effect on a continuous beam as an externally imposed deformation, and cause significant redistribution of bending moments possibly leading to flexural cracking (see Fig. 3.4(b)).

Differential temperature or shrinkage in a continuous reinforced-concrete beam can also cause a significant change to the bending moment distribution leading to flexural cracking. Examples of when this might need to be considered are a floor exposed to direct sunlight or a precast reinforced-concrete beam with a cast-insitu topping slab. Without the supports present, the change in curvature due to the effects of temperature or shrinkage would not cause the bending moment distribution to change. However, under the influence of gravity and applied loads, the beam deformation is restrained by the supports whereby the beam is considered to be subjected to *restrained deformation* (see Fig. 3.4(c)).

Direct tension is not normally associated with beams, which are considered to be flexural elements. However, for the purposes of this design booklet, the design of tension elements is also addressed. An example of such an element is a tension web or chord of a truss (see Fig. 3.4(d)). The tensile force in such an element normally arises principally from direct loading of the truss. An example of when a beam might be in a state of tension is if it is lightly loaded, and effectively restrained at both ends from moving longitudinally by braced walls. This is a classic case of *restrained deformation* arising due to the effects of concrete shrinkage (see Fig. 3.4(e)). Restraining the contraction of a beam subjected to a drop in average temperature also causes a resultant tensile force to develop.

It is beyond the scope of this design booklet to describe the methods of analysis that are required to calculate the change in design action effects that results for each of the actions described above. Standard computer programs that analyse redundant structures can be used to account for the effects of direct loading (whether causing tension or flexure), and imposed deformations such as support settlement. They can also be used to account for the effect that restrained deformation at supports has on the flexural action that is caused by differential temperature or differential shrinkage. However, the analysis of beams subjected to tension as a result of restrained deformation is not straightforward, and simplifying assumptions normally need to be made, e.g. [12]. In this latter case, the onset of cracking causes a significant reduction in axial stiffness and therefore tensile restraining force when concrete shrinkage is involved (see Section 7.4).

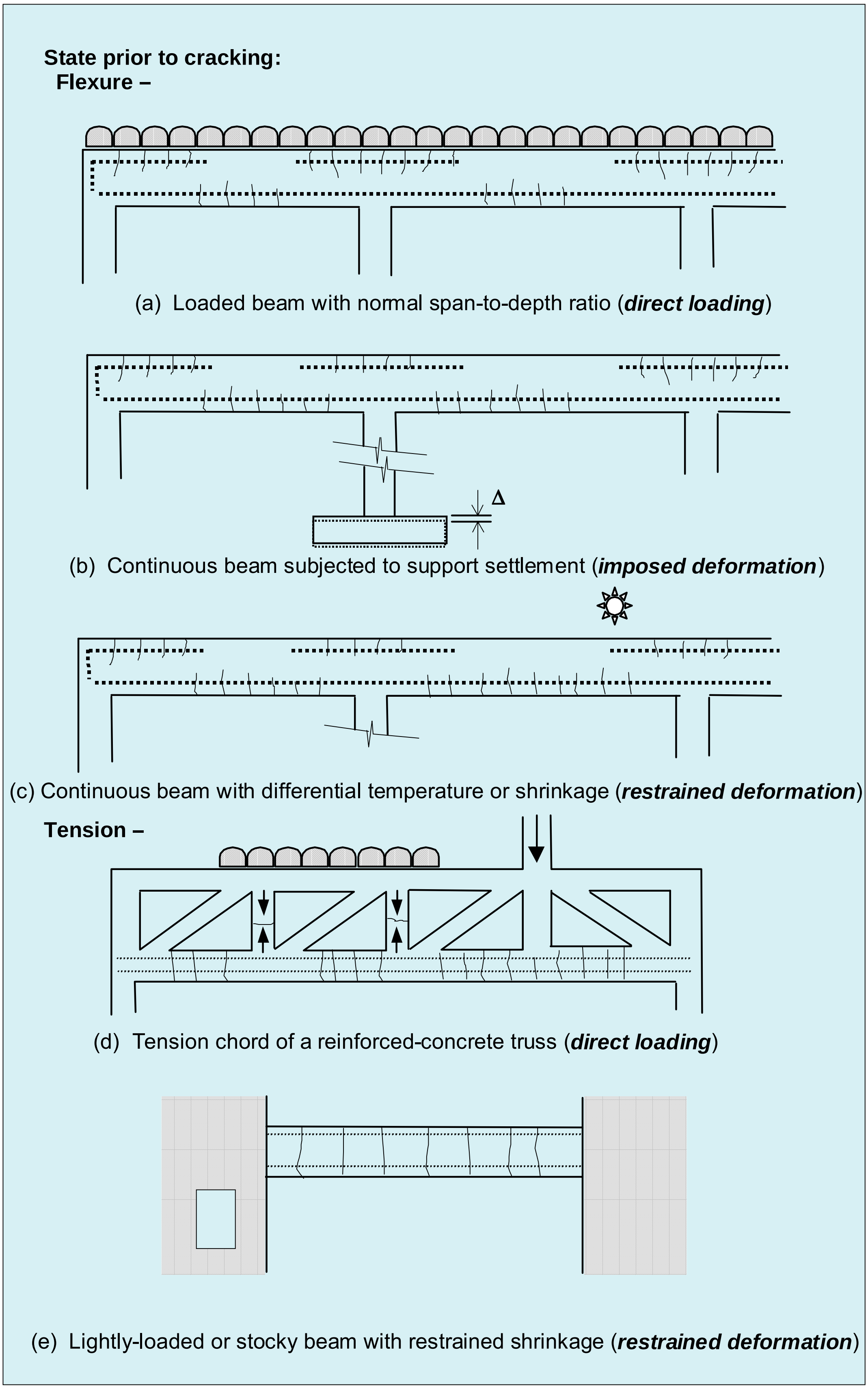


Figure 3.4 Examples of Beams Deemed to be in Flexure or Tension under Different Actions

3.5 Crack Width Calculations in Accordance with Eurocode 2

3.5.1 Introduction

Eurocode 2 provides engineers with two alternative methods for controlling cracking in reinforced-concrete beams. These are as follows.

(i) Calculation of Crack Widths (Clause 4.4.2.4) – formulae are provided for crack width calculations which apply to both beams and slabs for a range of design situations; or

(ii) Control of Cracking without Direct Calculation (Clause 4.4.2.3) – a simplified design method is allowed, the rules for which have been derived using the crack width formulae. Minimum reinforcement areas are determined, and limits placed on bar diameter and bar spacing.

The design method involving crack width calculations is briefly addressed in the following Sub-sections. Some changes have been made to the terminology to suit this design booklet, while some design situations covered in Eurocode 2 have also been omitted for brevity, e.g. references to plain bars which are not permitted as main reinforcement in Australia.

Simplified design in accordance with Eurocode 2 is covered in Section 3.6, which forms the basis for the new design rules proposed for inclusion in AS 3600-2000. Despite the fact that they have not been proposed for incorporation in AS 3600-2000, the crack width formulae are presented here to provide engineers with the opportunity to understand the background to the simplified method. Moreover, the derivation of the acceptable values of bar diameter and bar spacing as a function of maximum steel stress is also presented in Section 3.6.

The fundamental principles behind the design approach adopted in Eurocode 2 are stated in Clause 4.4.2.1(9) as follows:

(i) a minimum amount of bonded reinforcement is required;

(ii) yielding of the reinforcement must not occur during crack formation (see Section 3.7.5); and

(iii) crack control is achieved by limiting (the steel stress depending on) bar spacing and/or bar diameter.

3.5.2 Derivation of Crack Width Formulae

Beeby and Narayanan [11] can be referred to for a more detailed account of the derivation of the crack width formulae in Eurocode 2.

Similar in principle to Eq. 3.3.2.1(6), it can be written that:

$$w_k = \beta s_{crm} \varepsilon_{sm} \qquad 3.5.2(1)$$

where – w_k is the design crack width, which is a characteristic value with only a 5% probability of being exceeded;

β is a factor that relates the mean crack width in tests to the design value, e.g. it equals 1.7 for cracking due to direct loading;

s_{crm} is the average final crack spacing; and

ε_{sm} is the average difference in strain between the steel and the concrete including the effects of bond stress, tension stiffening, concrete shrinkage, etc.

Concerning crack spacing, in the first instance Eq. 3.3.2.1(4) is used to estimate the transfer length. Then the average crack spacing is assumed to equal 1.5 times this value, i.e. the average of Eqs 3.3.2.1(1) and 3.3.2.1(2). As an initial estimate for s_{crm}, it follows from Eq. 3.3.2.1(4) that:

$$s_{crm} = 1.5 \frac{d_b f_t}{4 \tau_m p_s} \qquad 3.5.2(2)$$

This can be rewritten as:

$$s_{\text{crm}} = \frac{k_1 d_\text{b}}{4 p_\text{s}} \qquad 3.5.2(3)$$

where the constant k_1=1.5f_t /τ_m. As mentioned in Section 3.3.2.1, a representative value of average bond stress for deformed bar is τ_m=5.5 MPa, and a minimum value of concrete tensile strength of f_t=3.0 MPa is recommended in Eurocode 2 (see Section 3.5.3), which gives k_1=0.82, while a value of 0.8 is specified in Eurocode 2.

It follows from above that Equation 3.5.2(3) applies to elements subjected to pure tension. It was further modified for inclusion in Eurocode 2 to cover both of these cases. The average final crack spacing for beams subjected dominantly to flexure or tension, s_crm (in mm), can be calculated from the equation:

$$s_{\text{crm}} = 50 + \frac{k_1 k_2 d_\text{b}}{4 p_\text{r}} \qquad 3.5.2(4)$$

where – k_1 is a factor that takes account of the bar bond properties, and

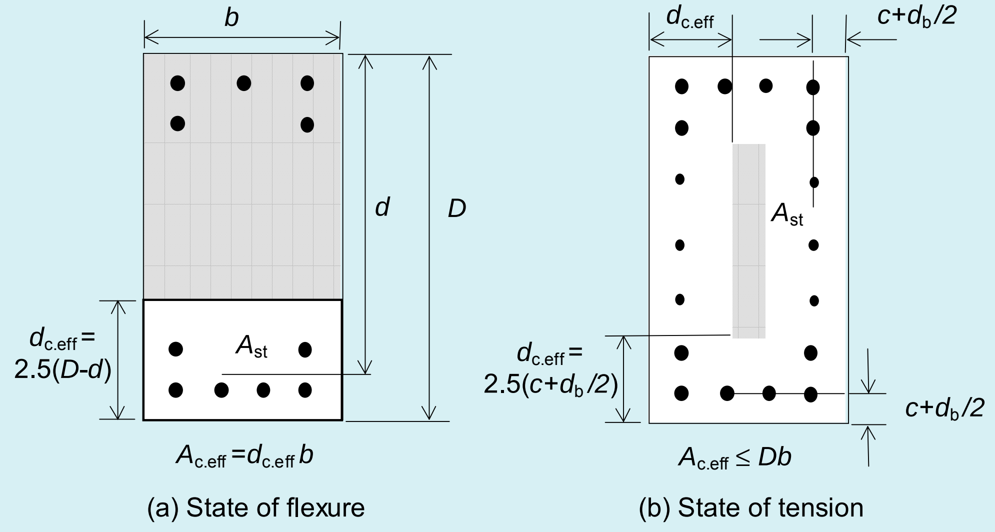
=0.8 for deformed bars (see above);

k_2 is a factor that takes account of the form of the stress distribution, thus allowing elements in a state of flexure as opposed to tension to also be designed, and

=0.5 for pure bending and 1.0 for pure tension, while intermediate values can be calculated for eccentric tension on the basis of a cracked section; and

p_r is the effective reinforcement ratio, A_st /$A_\text{c.eff}$, where A_st is the cross-sectional area of reinforcement within the effective tension area of concrete, $A_\text{c.eff}$ (see Fig. 3.5).

The first term of 50 mm was added to include the effect of concrete cover, which has been shown to have a direct effect on crack spacing. This was not explicitly included in the formula since it was felt that it might encourage engineers to minimise cover when designing for crack control. However, this does not appear to be entirely logical since designers will normally only specify the minimum cover necessary for durability. Moreover, for beams the cover to fitments controls the cover to the bars. Omitting cover as a variable more likely means that the design engineer will not be aware of a potential crack width problem in cases when large covers occur. A secondary beam framing into a primary beam incorporating large diameter bars placed above the top bars of the secondary beam, is an example of when the cover might well significantly exceed the minimum.



Notes (issues not addressed in Eurocode 2):

1. For the top flange of an L-beam or T-beam in tension under a state of flexure, replace b in (a) with b_{eff}. It is also necessary to ensure that $d_{c.eff} \leq D_s$.
2. For a state of tension, it is necessary to ensure that $d_{c.eff} \leq b/2$. For an L-beam or T-beam, the same principle for calculating $A_{c.eff}$ as shown in (b) can be used.

Figure 3.5 Definition of Effective Concrete Area, $A_{c.eff}$, for Beams

The average strain, ε_{sm}, is calculated at the section being considered as:

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left[1 - \beta_1\beta_2\left(\frac{f_{sr}}{f_s}\right)^2\right] \qquad 3.5.2(5)$$

where – f_s is the stress in the tension steel under the serviceability condition being considered, calculated on the basis of a cracked section, and equals f_{sr} in the case of restrained deformation giving rise to a state of tension (since then ε_{sm} must equal zero, at least for β_2=1.0);

f_{sr} is the stress in the tension steel under the relevant condition that just causes the tensile strength of the concrete to be reached, calculated on the basis of a cracked section;

β_1 is a factor that accounts for the bond properties of the reinforcement,

=1.0 for deformed bars; and

β_2 is a factor that accounts for repeated stressing of the bars,

=1.0 if the bars are only stressed once (which is not really relevant to design), or

=0.5 for repeated stressing (the normal design situation).

It is assumed in the derivation of Eq. 3.5.2(5) that the reinforcement is completely unbonded next to each crack, and fully bonded in the middle region between adjacent cracks. The proportion of the distance between adjacent cracks, i.e. crack spacing, over which the reinforcement is assumed to be unbonded equals the term in the square brackets in Eq. 3.5.2(5). In the extreme, if this term equals zero, then the unbonded length equals zero and consequently ε_{sm} equals zero. This corresponds to behaviour prior to cracking. Equally, if this term equals a maximum value of 1.0, then the bar would be unbonded over its entire length resulting in ε_{sm}=f_s/E_s. Equation 3.5.2(5) obviously has an empirical basis.

3.5.3 Minimum Reinforcement

Eurocode 2 requires that a minimum area of bonded reinforcement must be provided in beams subjected to restrained deformation where a state of tension is induced. This restraint might occur in combination with other actions. The steel must not yield while the cracks develop.

The equation for calculating this minimum area has been derived assuming equilibrium between the tensile forces in the steel and the concrete. It is similar in form to Eq. 3.3.2.2(2) except that, presumably for simplicity, the tensile force in the steel in the uncracked region has been ignored. Owing to this unconservative approximation, it seems appropriate to ignore the loss of area of concrete due to the steel reinforcement when applying this rule. The minimum area, $A_{st.min}$, is given by:

$$A_{st.min} = k_3 k_4 f_t A_{ct} / f_s \qquad 3.5.3(1)$$

where – k_3 is a factor that allows for the effect of non-uniform self-equilibrating stresses, e.g. due to differential shrinkage or temperature,

=0.8 for restrained deformation where a state of tension is induced, and may have other values for other situations (see Section 3.6.1);

k_4 is a factor that takes account of the stress distribution at the section of concern immediately prior to cracking,

=1.0 for restrained deformation where a state of tension is induced, and may have other values for other situations (see Section 3.6.1);

f_t is the (mean value of the) tensile strength of the concrete at the critical time when the cracks might occur, and a value of 3.0 MPa is recommended in Eurocode 2 for normal use;

A_{ct} is the area of concrete in the tensile zone at the section of concern prior to cracking, and for the reason explained above, the cross-sectional area of the tension steel should not be subtracted from the gross area of the section in tension when calculating A_{ct}; and

f_s is the maximum stress permitted in the reinforcement immediately after the formation of the crack.

Concerning the value of f_s in Eq. 3.5.3(1), it must not exceed the nominal yield strength of the reinforcement, f_{sy}. As explained above, this requirement is intended to ensure that multiple cracks can form. However, it might be necessary for this stress to be reduced to less than f_{sy} in order to achieve acceptable crack widths. Equation 3.5.2(5) in association with Eq. 3.5.2(1) can be used to determine if this is necessary. Alternatively, a value of f_s can be estimated using the simplified design method described in the next Section.

It should be noted that for a state of tension, the minimum area of steel, $A_{st.min}$, determined using Eq. 3.5.3(1) will need to be distributed in the top and bottom faces of the beam, and also possibly in the sides.

3.6 Simplified Design Method in Eurocode 2

3.6.1 Minimum Reinforcement

It is a basic requirement of the design rules in Eurocode 2 for controlling cracking without requiring direct calculation, that the minimum area of reinforcement given by Eq. 3.5.3(1) is placed at the cross-section being designed. Since elements in a state of flexure must naturally be considered, the use of Eq. 3.5.3(1) needs to be broadened by the following additional description of some of its terms.

With reference to Eq. 3.5.3(1), it can be further written that:

k_3 =1.0 for imposed deformation such as from support settlement; and

k_4 =0.4 for pure bending, although a general equation is derived below for determining this value more accurately if desired (see Eq. 3.6.1(5)). Eurocode 2 does not provide values of k_4 for stress states other than pure bending (k_4=0.4) or pure tension (k_4=1), which is discussed at the end of this Sub-section.

For pure bending, immediately after the first crack is induced (assuming that the bending moment at the critical section does not reduce), it can be written that:

$$f_s = \frac{M_{cr}}{A_{st.min} z} \quad \text{3.6.1(1)}$$

where z is the lever arm of the internal force couple.

However, immediately prior to cracking:

$$M_{cr} = f_t Z_t \quad \text{3.6.1(2)}$$

where Z_t is the section modulus on the tension side of the uncracked section.

Substituting Eq. 3.6.1(1) into Eq. 3.6.1(2) and rearranging gives:

$$A_{st.min} = \frac{f_t Z_t}{f_s z} \quad \text{3.6.1(3)}$$

From Eq. 3.5.3(1), with k_3 omitted it can be written that:

$$A_{st.min} = k_4 f_t A_{ct} / f_s \quad \text{3.6.1(4)}$$

Substituting Eq. 3.6.1(3) into Eq. 3.6.1(4) and rearranging finally gives a general equation that applies for pure bending:

$$k_4 = \frac{Z_t}{A_{ct} z} \quad \text{3.6.1(5)}$$

Considering a simple rectangular beam, ignoring the presence of the reinforcement when calculating Z_t and A_{ct}, i.e. $Z_t=bD^2/6$ and $A_{ct}=bD/2$, and assuming z=0.8D gives k_4=0.42, which explains the value of 0.4 in Eurocode 2.

Concerning the calculation of minimum steel for stress states arising from combined tension and bending, unfortunately Eurocode 2 does not provide values of k_4 for this purpose. Deriving values along the lines illustrated above is not straightforward, even for a simple rectangular section that may need to be reinforced in both faces. This is because an iterative calculation is required to calculate the stress in the steel for the cracked state when some compression is in the concrete [12]. Beeby and Narayanan [11] discuss this issue, and provide a chart (Fig. 8.35 therein) by which linear interpolation can be used to calculate the minimum area of steel in each face of a rectangular beam. Their chart is based on the assumption that the steel in each face is equally stressed for the fully cracked state with no compression present.

In view of this situation, a simple procedure for determining a value for k_4 is proposed as follows.

1. Determine whether the section is in a state of tension or flexure prior to the onset of cracking according to Section 3.4.
2. For a state of tension, assume k_4=1.0.
3. For a state of flexure, assume that k_4=0.6.

(Note: the value of k_4=0.6 approximately corresponds to the limit of a singly-reinforced section.)

Alternatively, the minimum reinforcement can be calculated more accurately using the following procedure. In this case, it is not necessary to use Eq. 3.5.3(1) directly.

1. Calculate the design action effects at the serviceability limit state, M_s^* and T_s^*, where M_s^* is the design bending moment, and T_s^* is the design tensile force assumed to act through the centroid of the uncracked section. Calculate the eccentricity of T_s^* relative to the centroid as $e_T = M_s^* / T_s^*$.

2. Based on an uncracked section, and assuming a maximum stress in the extreme tensile fibre of the concrete equal to f_t, calculate a relationship for the stress in the steel, f_s, corresponding to a bending moment and tensile force acting such that the eccentricity equals e_T. It is assumed that all of the tension steel is positioned so that it is at the same stress.

3. The minimum area of steel, $A_{st.min}$, can then be calculated for an appropriate value of f_s as defined for Eq. 3.5.3(1).

However, this procedure just described will not normally be used by designers, and is given more for completeness.

3.6.2 Crack Width as a Function of Bar Diameter or Bar Spacing

The simplified design rules in Eurocode 2 will normally be used by design engineers, thus avoiding direct calculation of crack width. They have been derived using the crack width formula Eq. 3.5.2(1) in parametric studies, as will be explained below.

Limits are placed on bar diameter and bar spacing to ensure that crack widths will not generally exceed 0.3 mm for reinforced-concrete beams. Specifically, it is stated that:

(i) for cracking caused dominantly by restraint (herein interpreted to apply to beam sections in a state of tension or flexure – see Note 1), the maximum diameters of deformed bars given in Table 3.6.2(1) are not to be exceeded where the steel stress, f_s, is the value obtained immediately after cracking (i.e. f_s in Eq. 3.5.3(1)); and

(ii) for cracking caused dominantly by loading (herein interpreted to apply to beam sections in a state of flexure – see Note 2), either the maximum diameters of deformed bars given in Table 3.6.2(1) or their maximum spacings given in Table 3.6.2(2) must be complied with. (Thus, in this case it is not necessary to satisfy both tables.)

Note 1: As shown in Fig. 3.4, restrained deformation can give rise to a state of flexure (Fig. 3.4(c)) or tension (Fig. 3.4(e)). It will be seen below that Table 3.6.2(1) was derived for a beam in pure bending. It has been assumed that the table also applies to beams in tension as indicated in Eurocode 2. Therefore, it has been assumed that the table applies to beams in a state of tension or flexure (see Section 5.3).

Note 2: Similarly, direct loading can also give rise to a state of flexure (Fig. 3.4(a)) or tension (Fig. 3.4(d)). It will be seen below that Table 3.6.2(2) applies to beams in either pure bending (column 2) or pure tension (column 3). However, it is not explained in Eurocode 2 how a designer should interpolate between these two extreme situations. It has been assumed that the values for pure bending are applicable for flexure (see Section 5.3).

It should be noted that in cases when only Table 3.6.2(1) is complied with, certain bar spacing requirements also apply (see Section 3.7). In addition, certain qualifications applying to Table 3.6.2(1) that are contained in Eurocode 2 have been omitted for simplicity here. They allow the maximum bar diameters to be increased depending on the tensile strength of the concrete and the overall depth of the beam. They are briefly discussed below.

For sections in a state of flexure, the steel stress is calculated under the quasi-permanent combination of loading, i.e. long-term serviceability condition. Beeby and Narayanan [11] state that this stress can be estimated by multiplying the design yield strength of the steel (taken as 500/1.15 in Eurocode 2) by the ratio of the quasi-permanent load to the design ultimate load. However, this assumes that (i) the steel area is controlled by strength; and (ii) that the same load combinations apply for strength and serviceability design. The designer should be aware of these assumptions, and the effect of any moment redistribution assumed at the strength limit state. This is discussed further in Section 4.1.

Table 3.6.2(1)
Maximum Bar Diameters

Steel stress (MPa)	Maximum bar diameter (d_b) (mm)
450	6
400	8
360	10
320	12
280	16
240	20
200	25
160	32

Table 3.6.2(2)
Maximum Bar Spacings

Steel stress (MPa)	Maximum spacing – pure bending (mm)	Maximum spacing – pure tension (mm)
360	50	-
320	100	-
280	150	75
240	200	125
200	250	150
160	300	200

The background to both of these design tables is briefly described as follows [10].

Firstly, consider Table 3.6.2(1). For a beam with a rectangular section subjected to bending, from Eq. 3.5.3(1) using k_3=1.0 (an appropriate value for bending – see Section 3.6.1), k_4=0.4, $A_{ct}=bD/2$ and $f_s=f_{sr}$, it follows that at the cracking moment:

$$f_{sr} = 0.2\frac{f_t}{\bar{p}} \qquad 3.6.2(1)$$

where $\bar{p} = \dfrac{A_{st}}{bD}$.

Substituting Eq. 3.6.2(1) into Eq. 3.5.2(5), and choosing β_2=0.5, which should normally be used for design, it follows that:

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left[1 - 0.5\left(\frac{0.2f_t}{\bar{p}f_s}\right)^2\right] \qquad 3.6.2(2)$$

Referring to the terms in Eq. 3.5.2(4) and Fig. 3.5, it follows that one can write $p_r = \dfrac{A_{st}}{A_{c.eff}}$ and $A_{c.eff}=2.5(D-d)b$, whereby:

$$p_r = \bar{p}\frac{D}{2.5(D - d)} \qquad 3.6.2(3)$$

Substituting Eq. 3.6.2(3) into Eq. 3.5.2(4) with k_1=0.8 and k_2=0.5 for pure bending gives:

$$s_{crm} = 50 + \frac{d_b(D - d)}{4D\bar{p}} \qquad 3.6.2(4)$$

Finally, substituting Eqs 3.6.2(2) and 3.6.2(4) into Eq. 3.5.2(1), choosing β=1.7 and transposing in terms of bar diameter, d_b, gives:

$$\max . d_b = \frac{4D\bar{p}}{(D - d)}\left[\frac{w_k}{1.7\frac{f_s}{E_s}\left\{1 - 0.5\left(\frac{0.2f_t}{\bar{p}f_s}\right)^2\right\}} - 50\right] \qquad 3.6.2(5)$$

This equation is plotted in Fig. 3.6 for: w_k=0.3 mm; f_s=280 MPa (corresponding to a 16 mm diameter bar, as indicated by the heavy dashed horizontal line); (D-d)=40 mm, f_t=2.5 MPa (as was assumed in the Eurocode 2 parametric study); and E_s=200 GPa. The overall depth of the beam, D, was 400 mm, noting that max. d_b reduces with D, whereby similar curves for deeper beams are higher than the one shown, while shallower beams give lower curves. The unfavourable effect for values of reinforcement ratio, $\bar{p}$, less than about 0.45% (i.e. when the curve in Fig. 3.6 falls below the heavy dashed horizontal line) was neglected in the study.

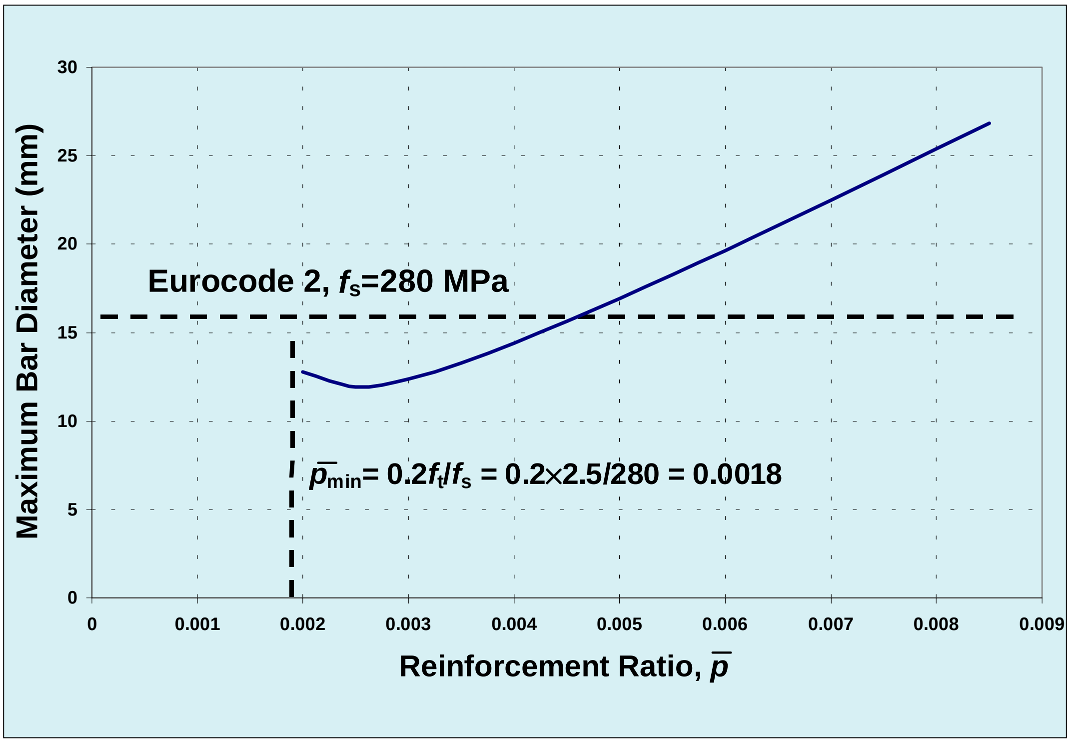


Figure 3.6 Derivation of Table 3.6.2(1) from Eurocode 2 (Pure Bending)

It follows from Fig. 3.6 that Table 3.6.2(1) may be unconservative for lightly-reinforced, shallow beams. For example, the curve in Fig. 3.6 can be lifted above the dashed line for d_b=16 mm by assuming w_k=0.4 mm. It also follows that the table is possibly overly conservative for deep, heavily-reinforced beams. In this regard, Eurocode 2 recognises that the maximum bar diameter increases approximately linearly with overall depth while cover remains constant, and allows a correction factor to be calculated to increase the maximum bar diameter. The maximum bar diameter can also be increased if the tensile strength of the concrete is greater than 2.5 MPa in design.

Despite these apparent refinements, it should be remembered that the variability between measured and predicted crack widths is high when using this design approach. The empiricism described in Section 3.5.2, particularly with regard to the calculation of ε_{sm} partly explains this. Therefore, using Table 3.6.2(1) directly without further refinements is probably all that is justified in practice, although crack width calculations are permitted by Eurocode 2. A reasonable means of overcoming the lack of conservatism for shallow, lightly-reinforced beams under direct loading could be to base the calculation of serviceability design action effects on short-term rather than long-term loading, i.e. use ψ_s instead of ψ_l.

Now, considering the maximum bar spacings in Table 3.6.2(2), transformation of Eq. 3.6.2(5) gives:

$$w_k = 1.7\frac{f_s}{E_s}\left[1-0.5\left(\frac{0.2f_t}{\bar{p}f_s}\right)^2\right]\left(50+\frac{d_b(D-d)}{4D\bar{p}}\right) \qquad 3.6.2(6)$$

Remembering that this equation applies for pure bending (k_2=0.5 in Eq. 3.5.2(4)), and assigning as before D=400 mm, (D-d)=40 mm, f_t=2.5 MPa, and calculating the steel reinforcement ratio approximately as:

$$\bar{p} = \frac{A_b}{s_b D} = \frac{\pi d_b^2}{4 s_b D} \qquad 3.6.2(7)$$

where A_b and s_b are the cross-sectional area and spacing of the bars, respectively.

For each value of steel stress, f_s, in Table 3.6.2(2), the crack width was calculated using Eq. 3.6.2(6) for each bar spacing in the middle column of the table, and for each bar diameter in Table 3.6.2(1). The results of these calculations are shown in Fig. 3.7, noting that any negative values of w_k calculated have been omitted because these are non-critical situations. It is apparent from Fig. 3.7 that designs based on Table 3.6.2(2) for pure bending give maximum crack widths of around 0.35 mm, and should generally be satisfactory. However, it should not be overlooked that by using Table 3.6.2(2), the value of the reinforcement ratio, $\bar{p}$, can be quite high, particularly if large diameter bars are used at high stresses and therefore close centres. This implies that solutions satisfying Table 3.6.2(2) may not necessarily be practical or economical. Insight into when the requirements in Table 3.6.2(2) for pure bending can be used as an alternative to those in Table 3.6.2(1) is given in Appendix E.

For the case of pure tension, the value of k_2 in Eq. 3.5.2(4) has increased from 0.5 to 1.0, i.e. doubled. It follows from the second term in Eq. 3.5.2(4) that if the crack width is to remain the same, the value of p_r/d_b needs to about double. However, similar to Eq. 3.6.2(7), it is a simple matter to show that $p_r \propto d_b^2/s_b$, or equivalently that $p_r/d_b \propto d_b/s_b$. It follows that to keep the crack width about equal requires the value of d_b/s_b to about double. Therefore, for the same bar diameter, the bar spacing should approximately halve. Although this is not always precisely true, it can be seen by comparing the second and third columns in Table 3.6.2(2) that this is a reasonable explanation for the difference between the maximum spacings for pure bending and pure tension.

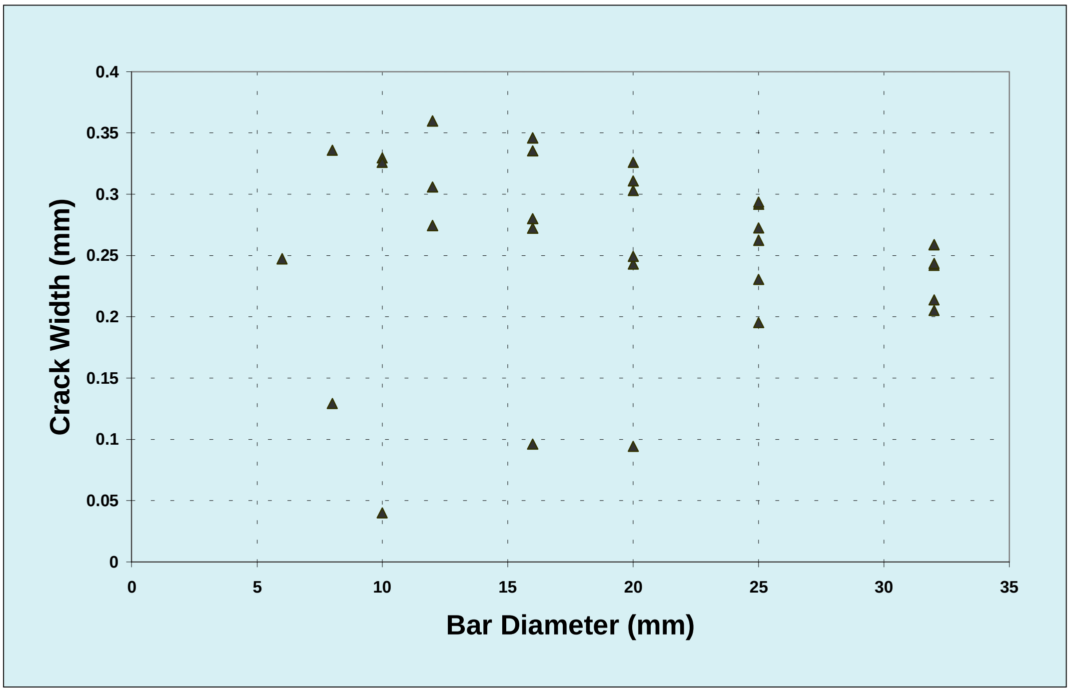


Figure 3.7 Derivation of Table 3.6.2(2) from Eurocode 2 (Pure Bending)

3.7 Other Relevant Design Requirements in Eurocode 2

3.7.1 Skin Reinforcement (Clause 4.4.2.3(4))

Beams with an overall depth of at least 1000 mm where the main longitudinal reinforcement is concentrated in only a small proportion of the depth require additional skin reinforcement to control cracking on the side faces of the beam.

3.7.2 Bundled Bars (Clause 5.2.7.1(2))

Bundled bars may be treated as individual bars for the purpose of crack control design. In a bundle, all the bars must have the same diameter, d_b, and characteristics (type and grade). In design, a bundle is replaced by a notional bar having the same cross-sectional area and the same centre of gravity as the bundle. The equivalent diameter of the notional bar shall be taken as $d_b\sqrt{n_b}$ (not to exceed 55 mm), where n_b is the number of bars in a bundle and is not to exceed 3. However, Eurocode 2 requires surface reinforcement consisting of mesh or small diameter bars placed outside the ligatures to be used to resist spalling when bundled bars are used in beams.

3.7.3 Minimum Reinforcement Percentage (Clause 5.4.2.1.1(1))

The cross-sectional area of the longitudinal tension reinforcement must not be less than that required to control cracking, nor less than $0.0015b_td$ where b_t is the mean width of the tension zone. For a T-beam with the flange in compression, b_t equals the width of the web.

3.7.4 Detailing in Monolithic Construction (Clause 5.4.2.1.2(1))

In monolithic construction, if simple supports are assumed in design, the end regions should be designed for a bending moment arising from partial fixity of at least 25 per cent of the maximum bending moment in the span.

3.7.5 Avoiding Yielding of Reinforcement (Clause 4.4.1.1(6-7))

An essential condition for the crack width formulae given in Section 3.5.2 to be valid is that the reinforcement remains elastic. The condition immediately after each new crack is formed is critical, since yielding at a crack may prevent further cracks from forming. Also, should yielding, (or more precisely, significant non-linearity along the stress-strain relationship) occur at a crack at any stage, then this crack will become excessively wide and render the structure unserviceable.

Stresses in the steel under serviceability conditions that could lead to inelastic deformation of the reinforcement should be avoided as this will lead to large, permanently open cracks. It is assumed that this requirement will be satisfied, provided under the rare combination of loads (i.e. full, unfactored dead and live loads) that the tensile stress in the reinforcement does not exceed $0.8f_{sy}$. The intention here is that the effect of actions ignored in design (such as restrained deformation) will not be enough to cause the steel to yield. When a major part of the stress is due to restrained deformation, e.g. restrained shrinkage, then a maximum stress of f_{sy} is deemed acceptable in Eurocode 2.

4. DESIGN APPROACHES

4.1 General

The simplified design rules for crack control in Eurocode 2 give rise to a minimum area of reinforcement and a limitation on bar diameter or bar spacing, depending on the magnitude of the steel stress. They have been proposed for inclusion in AS 3600-2000. This has principally come about with the move from 400 MPa to 500 MPa grade reinforcing bars.

Design for crack control must be considered very carefully if the full benefit of this increase in steel strength is to be obtained in beams. This benefit can simply be measured by a reduction in the area of tension steel. Bending strength normally governs the amount of longitudinal tension steel at critical sections of reinforced-concrete beams designed to AS 3600-1994. If this remains the governing criterion when 500PLUS Rebar is used, then the saving of main steel can be as much as 20 per cent due to change in f_{sy} alone. It will also be shown that further benefits can arise by requiring fewer of the higher strength bars, which can reduce the need for multiple layers and improve section efficiency. A principal objective of the design approach presented in this booklet is to achieve, whenever possible, this full benefit when designing beams. This will require the design engineer to thoroughly understand the effect that bar spacing and bar diameter can have on the maximum allowable steel stress, while still keeping crack widths to an acceptable level.

In accordance with Clause 8.6.1 of AS 3600-1994, the centre-to-centre spacing of bars near the tension face of a beam must not exceed 200 mm. It follows from Table 3.6.2(2) that for pure bending, the steel stress under serviceability conditions generally should not exceed 240 MPa if crack widths are to be kept below about 0.3 mm. This may only be possible for 500 MPa bars if the steel area is governed by serviceability. For example, it follows from the discussion in Section 3.6.2 that the stress in the steel under serviceability conditions (ignoring moment redistribution at the strength limit state) can be approximately calculated using the equation:

$$f_s = \phi f_{sy} \frac{G + \psi_s Q}{1.25G + 1.5Q} \qquad 4.1(1)$$

where ϕ if the capacity factor for bending (normally 0.8) and ψ_s is the short-term live load factor (see Section 3.6.2 for explanation in choosing short-term rather than long-term loading). Therefore, the serviceability steel stress, f_s, can approximately vary between about 0.4 and 0.65 f_{sy}, depending on the ratio of dead and live loads. Thus, f_s can vary from 160 to 260 MPa for f_{sy}=400 MPa, and from 200 to 325 MPa for f_{sy}=500 MPa.

If moment redistribution is taken into account, Eq. 4.1(1) can be generalised further. Let η equal the degree of moment redistribution away from a critical moment region assumed during strength design. It will be assumed that η=0 implies no redistribution, and η=1 means 100% redistribution, noting that at least for elastic design, AS 3600 limits the absolute value of η to 0.3. Then it can be approximately written that:

$$f_s = \phi f_{sy} \frac{G + \psi_s Q}{1.25G + 1.5Q} \frac{1}{(1-\eta)} \qquad 4.1(2)$$

If η=0.3, it is clear that the serviceability steel stress, f_s, can be more than 40 percent higher than the values just cited, conceivably reaching 0.93 f_{sy} or 465 MPa for 500 MPa steel.

It is clear that a maximum bar spacing of 200 mm can no longer be relied upon to control cracking in beams with 500 MPa steel, where advantage has been taken of the higher strength, and the area of steel reduced accordingly. One approach that could have been taken when writing the new code rule would have been to simply reduce the maximum bar spacing to about 100 mm (see Table 3.6.2(2)), but this was felt to be too restrictive. In any case, it was felt design engineers need to be much more knowledgeable about the subject, because the rules in AS 3600-1994 were over-simplified and there are many real cases of excessively cracked structures.

4.2 Simplified Design Rules vs Crack Width Calculation

It is explained in the commentary to AS 3600-1994 that the calculation of crack widths can be used as an alternative procedure to the specific detailing rules for controlling cracking. The conditions when this more detailed design work might be required are not explained. However, it is stated in the commentary that designers should aim to minimise the cover and distance between bars to control flexural crack widths. It follows that if cover or bar spacing was larger than normal, then designers should have been cautious and calculated crack widths.

Reference is made in the commentary to AS 3600-1994 to accepted procedures in the American and British Standards: an earlier version of the current ACI 318 [17] (although the method has remained largely unchanged); and BS 8110: Part 2 [18]. It can be assumed that the design procedure in Eurocode 2 will eventually supersede that in BS 8110: Part 2, when Eurocode 2 becomes more widely adopted. Advice is not given in AS 3600-1994 for controlling cracking in beams due to direct tension that develops from longitudinal restraint.

Designers generally prefer not having to calculate crack widths. This is partly because these calculations can be iterative in nature. The simplified method in Eurocode 2 described in Section 3.6 provides reasonable solutions directly without iteration. However, it does require steel stresses under serviceability conditions to be calculated, which merits the use of computer software. Computer software also allows multiple solutions to be considered without requiring tedious hand calculations. Computer program 500PLUS-BCC™ has been developed for this purpose and is described in Section 6.

4.3 Flowcharts for Simplified Design Approaches

AS 3600 - 1994

The rules for crack control design in AS 3600-1994 are stated in Section 5.2. The design approach is very straightforward because bar spacing is the only criterion to consider. It is presented in the flowchart in Fig. 4.1.

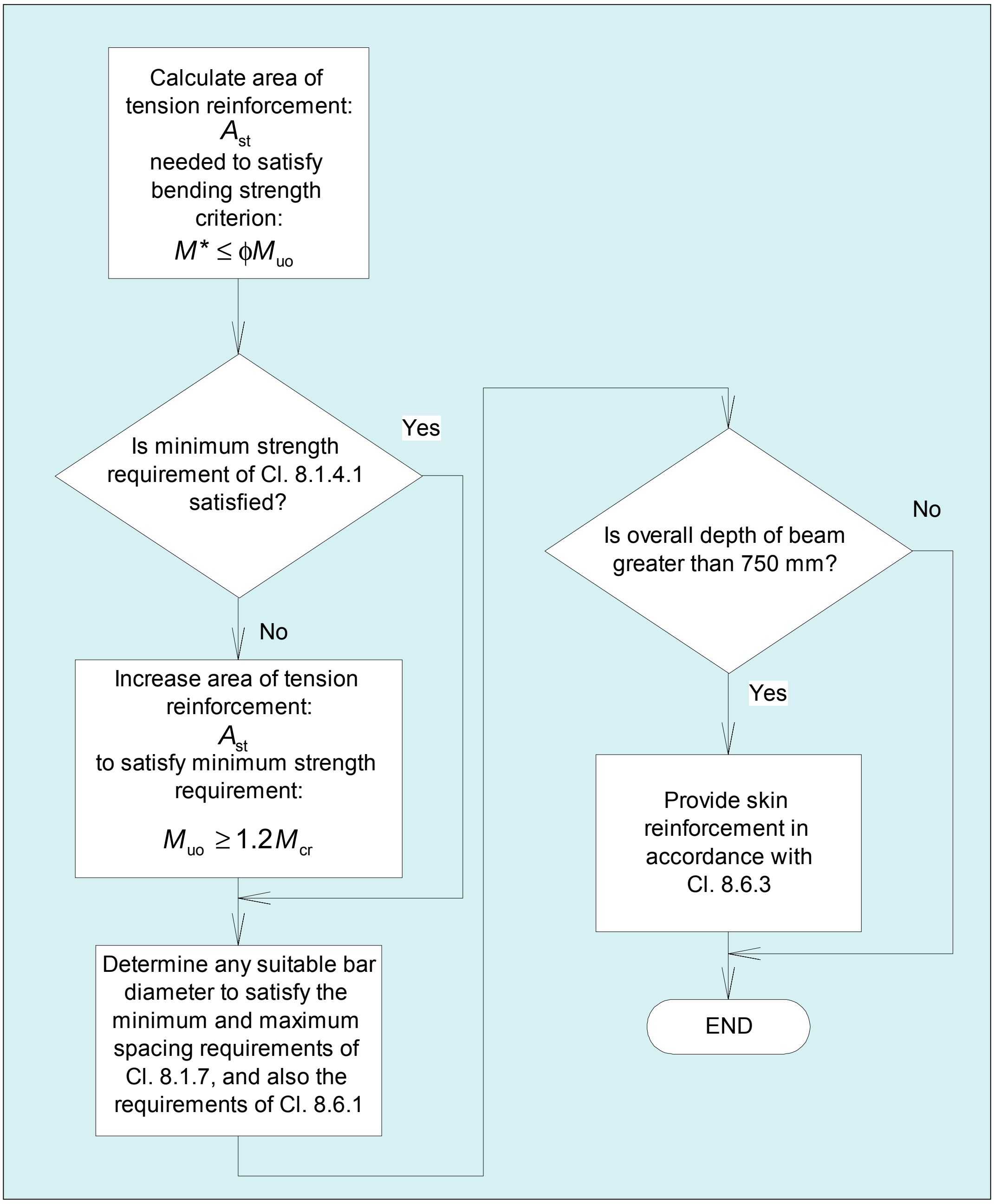


Figure 4.1 Flowchart of Simplified Design Approach in AS 3600-1994

AS 3600 - 2000

The rules for crack control design proposed for inclusion in AS 3600-2000 are stated in Section 5.3. The design approach is explained in the flowcharts in Figs 4.2, 4.3, 4.4 and 4.5.

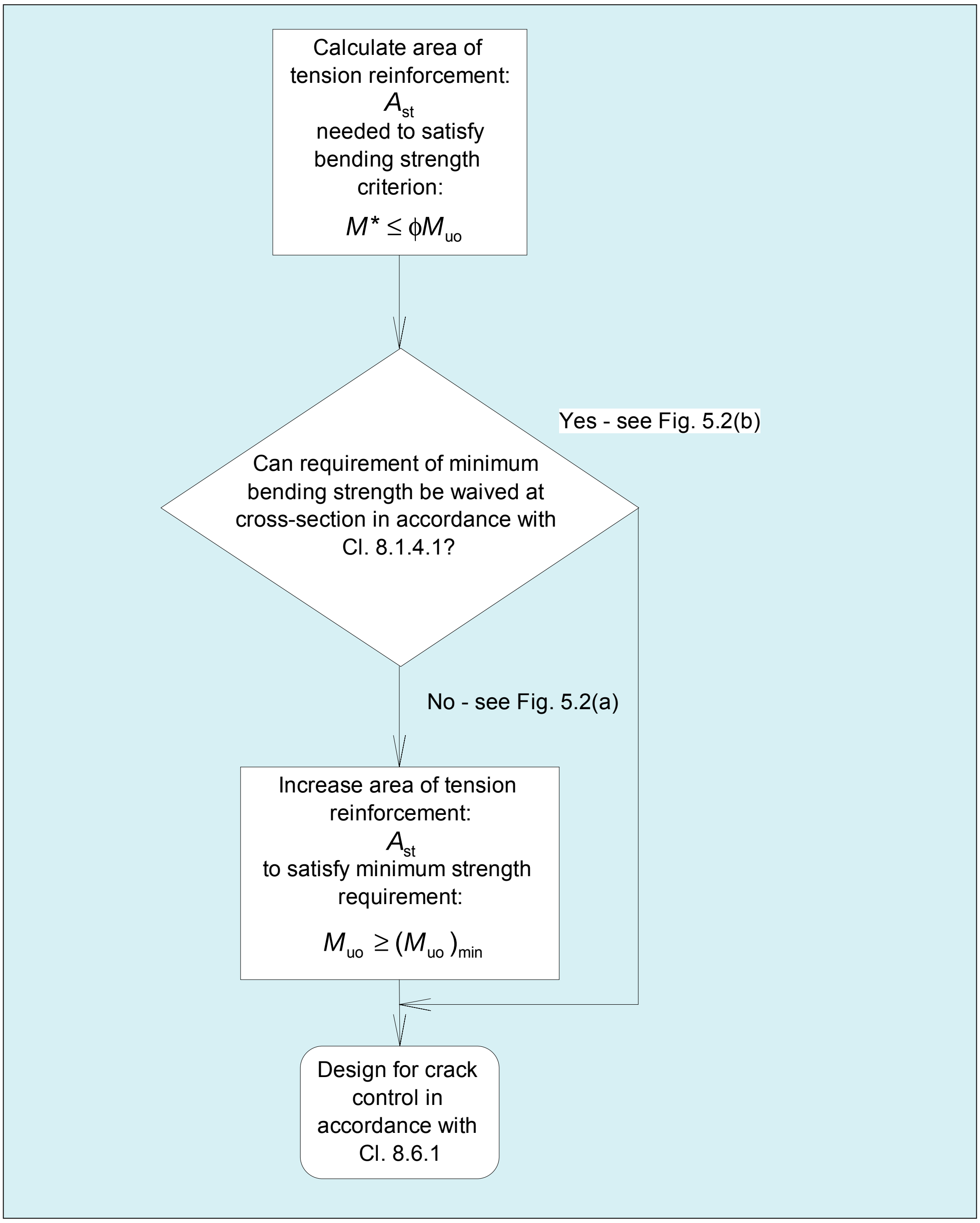


Figure 4.2 Flowchart of Simplified Design Approach in AS 3600-2000 – Part A: Minimum Strength Requirement for Beams in Bending

Note: For a tension member, the area of tension reinforcement A_{st} can also be calculated initially at the strength limit state. This value can then be used in the serviceability calculations described in Figs 4.3 to 4.5 to determine whether it needs to be increased.

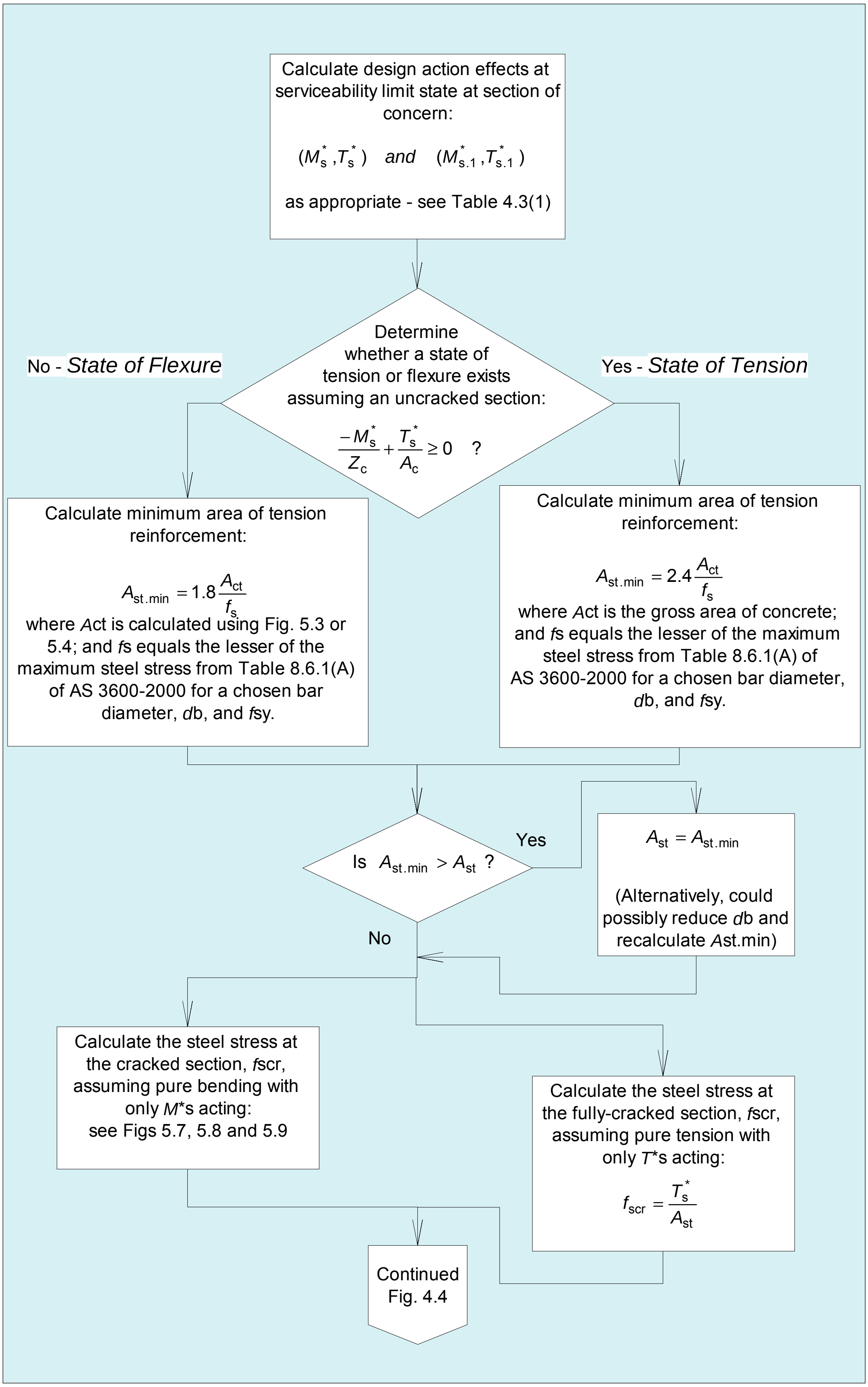


Figure 4.3 Flowchart of Simplified Design Approach in AS 3600-2000 – Part B: Crack Control for Tension and Flexure (cont.)

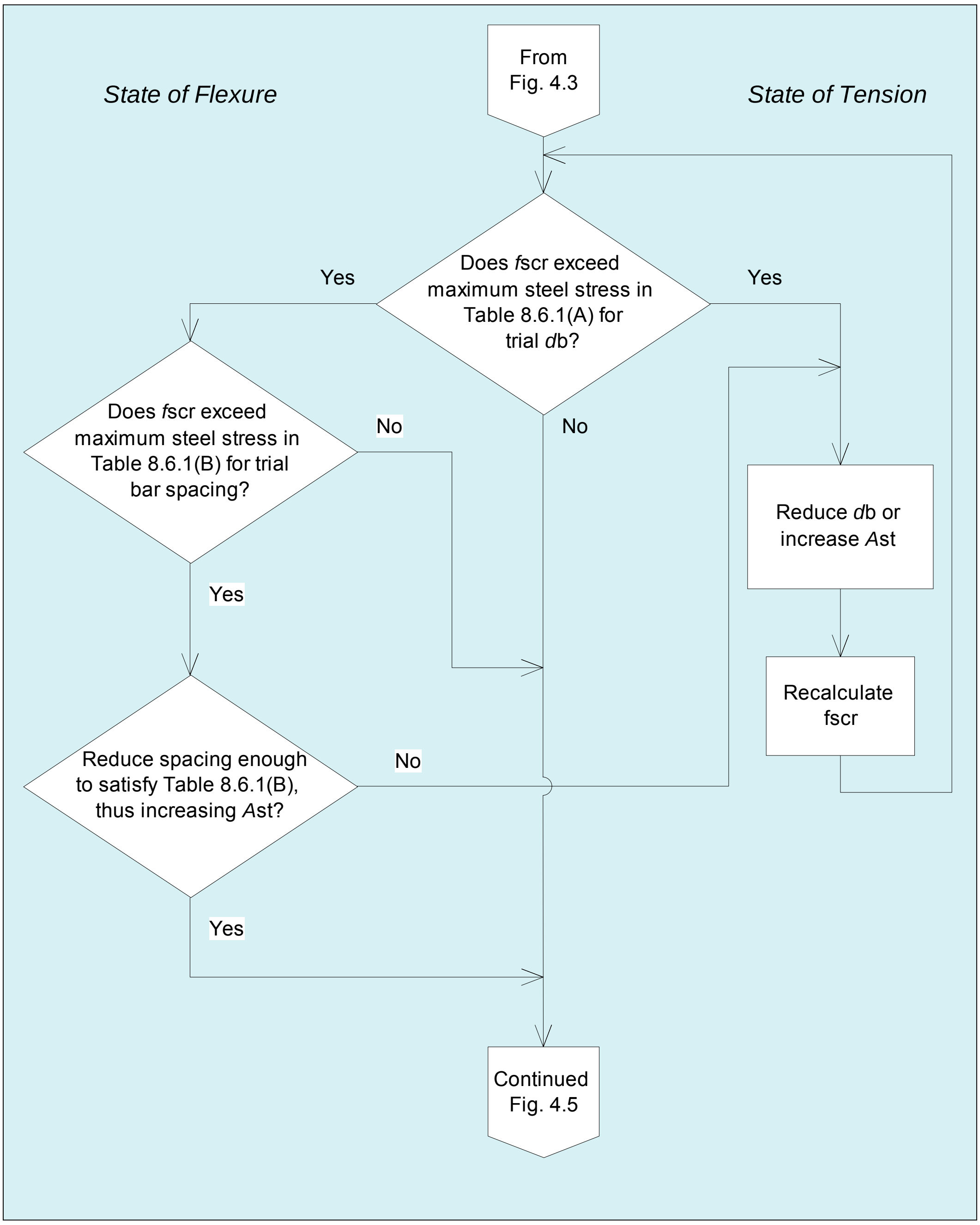


Figure 4.4 Flowchart of Simplified Design Approach in AS 3600-2000 – Part B: Crack Control for Tension and Flexure (cont.)

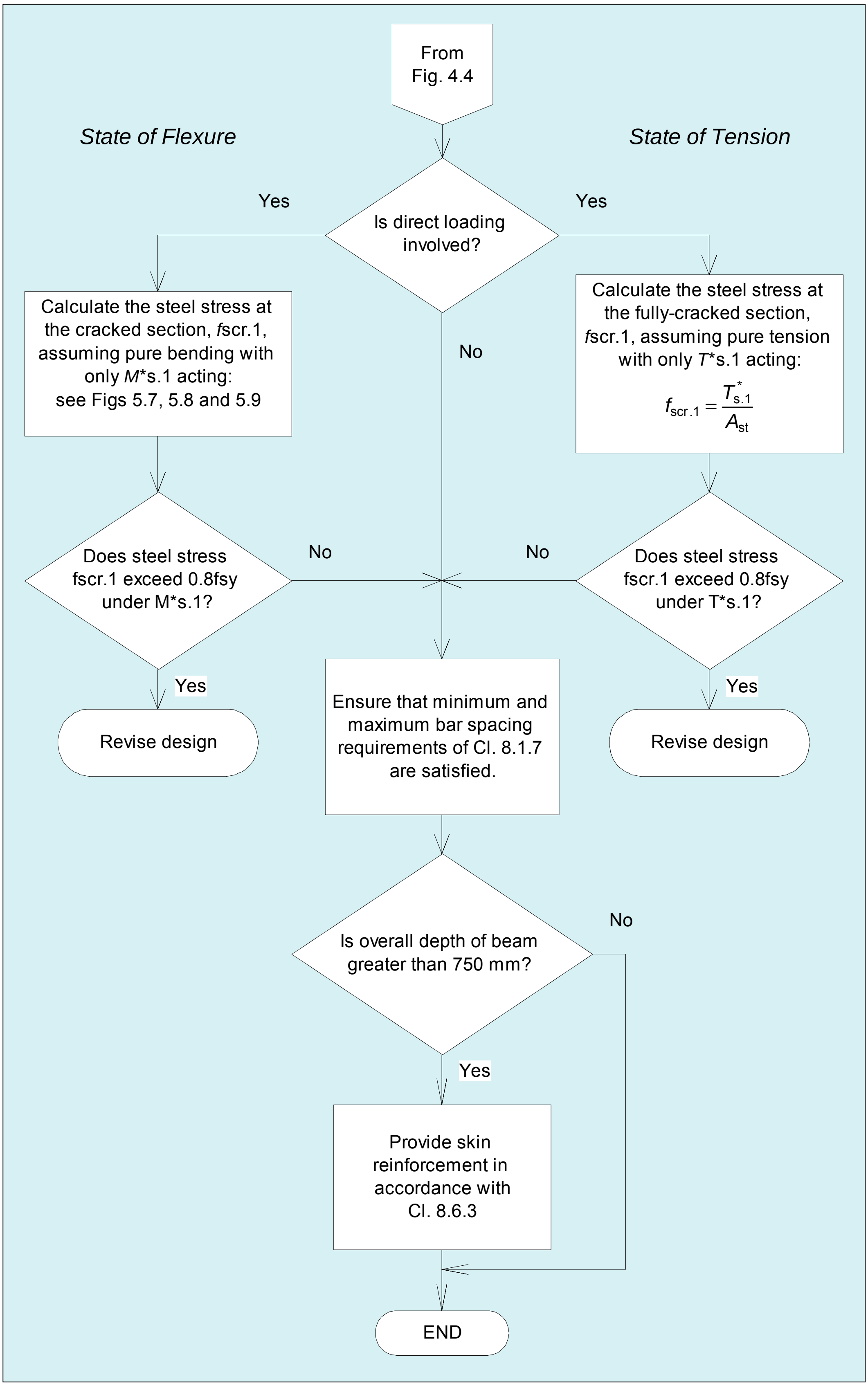


Figure 4.5 Flowchart of Simplified Design Approach in AS 3600-2000 – Part B: Crack Control for Tension and Flexure

Table 4.3(1)

Design Action Effects at Serviceability Limit State for Crack Control Design

(Refer to Figs 4.3 and 4.5)

State	Design action (see Fig. 3.4)	Design action effects	
		Uncracked section (Note 1)	Cracked section (Note 2)
Flexure	Direct loading	M_s^*, (T_s^*)	M_s^*, $M_{s.1}^*$ (Note 3)
"	Imposed deformation	M_s^*, (T_s^*)	M_s^*
"	Restrained deformation	M_s^*, (T_s^*)	M_s^*
Tension	Direct loading	(M_s^*), T_s^*	T_s^*, $T_{s.1}^*$ (Note 3)
"	Imposed deformation	(M_s^*), T_s^*	T_s^*
"	Restrained deformation	(M_s^*), T_s^*	T_s^* (Note 4)

Notes:

1. The symbol T is used generally in this design booklet for tensile force (see Notation).
2. In principle, both M_s^* and T_s^* should be known to determine whether a state of flexure or tension exists. However, normally a designer will make an assumption about the state of stress before undertaking an analysis to determine the design action effects. This will often negate the need to calculate both design action effects, with the designer only calculating the primary design action effect M_s^* or T_s^*. The secondary design action effect is indicated in parentheses.
3. The design action effect/s that act on the cracked section are used to calculate the stress in the steel under serviceability conditions. They are normally assumed to equal the design action effects that act on the uncracked section, which in turn may be calculated directly from the values determined for the strength limit state (see Eq. 4.1(2)). However, this is not always the case (see Note 4).
4. Under direct loading, the stresses in the steel under both $G+\psi_s Q$ (corresponding to M_s^* or T_s^*) and $G+Q$ (corresponding to $M_{s.1}^*$ or $T_{s.1}^*$) must be checked.
5. Under restrained deformation it may be necessary to calculate the design tensile force, T_s^*, after cracking, since cracking can significantly reduce the magnitude of this restraining force. For example, the reader is referred to Section 7.4 where it is shown that in the case of restrained shrinkage, T_s^* needs to be calculated for the long-term situation when the crack pattern is fully developed and shrinkage is effectively complete.

5. DESIGN RULES

5.1 General

The existing rules in AS 3600-1994 and proposed rules in AS 3600-2000[3] relating to the crack control design of beams are presented in this Section. In AS 3600-1994, it was deemed sufficient to limit the maximum spacing of the main tension bars, and their concrete cover. However, it is proposed in AS 3600-2000 that steel stresses under serviceability conditions will have to be calculated using elastic cracked-section theory, and then a suitable bar diameter or bar spacing chosen. Maximum cover will also be restricted. The equations needed to perform these stress calculations are provided in this Section as an aid to designers.

5.2 AS 3600 - 1994

The design rules in AS 3600-1994 relevant to crack control of beams in flexure are as follows. Background information to the rules can be found in the Commentary to AS 3600-1994.

Clause 2.4.4 Cracking (Clause 2.4, Design for Serviceability)

The cracking of beams or slabs under service conditions shall be controlled in accordance with the requirements of Clause 8.6 or 9.4 as appropriate.

Clause 8.1.4.1 General (Clause 8.1.4, Minimum Strength Requirements)

The ultimate strength in bending, M_{uo}, at critical sections shall be not less than 1.2 times the cracking moment, M_{cr}, given by-

$$M_{cr} = Z\,(f'_{cf} + P/A_g) + Pe \qquad 5.2(1)$$

where –

Z = section modulus of the uncracked section, referred to the extreme fibre at which flexural cracking occurs; and

f'_{cf} = characteristic flexural tensile strength of the concrete.

For the purpose of this Clause, the critical section to be considered for negative moment shall be the weakest section in the negative moment region (i.e. where $\phi M_{uo}/M^*$ is least).

For rectangular reinforced concrete sections, this requirement shall be deemed to be satisfied if minimum tension reinforcement is provided such that –

$$A_{st}/bd \geq 1.4/f_{sy} \qquad 5.2(2)$$

For reinforced T-beams or L-beams where the web is in tension, b shall be taken as b_w.

Clause 8.1.7 Spacing of reinforcement and tendons (Clause 8.1, Strength of Beams in Bending)

The minimum clear distance between parallel bars (including bundled bars), ducts and tendons shall be such that the concrete can be properly placed and compacted in accordance with Clause 19.1.3 (Handling, Placing and Compacting of Concrete). The maximum spacing of longitudinal reinforcement and tendons shall be determined in accordance with Clause 8.6 (Crack Control of Beams).

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[3] AS 3600-2000 is expected to be published this year by Standards Australia. The design rules proposed by the authors for possible inclusion in AS 3600-2000 are presented in Section 5.3. They are the result of further development work performed after the rules were first drafted, and released publicly in the Standards Australia Public Comment Draft DR 99193 CP, Amendment 2 to AS 3600. The clauses proposed in Section 5.3 can be compared directly with clauses with the same numbers in DR 99193 CP, which are reproduced in Appendix D. Any differences between the clauses in Section 5.3 and Appendix D reflect the authors' latest recommendations.

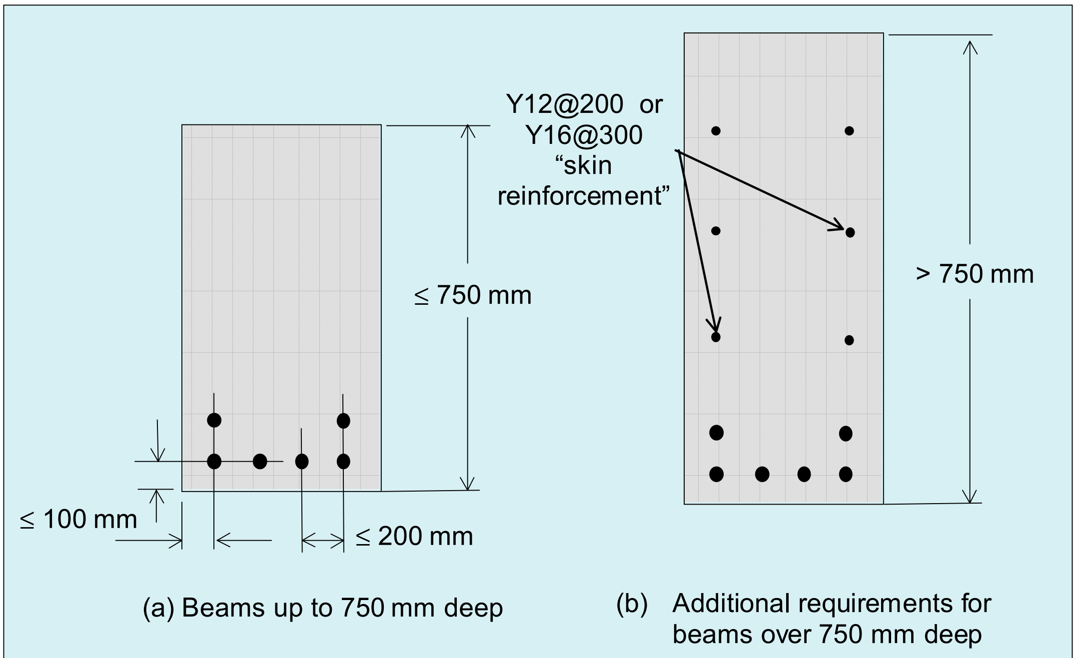
Clause 8.6.1 Crack control for flexure in reinforced beams (Clause 8.6, Crack Control of Beams)

Flexural cracking in reinforced beams shall be deemed to be controlled if the following requirements are satisfied (see Fig. 5.1(a)).

(a) The centre-to-centre spacing of bars near the tension face of the beam shall not exceed 200 mm.

(b) The distance from the side or soffit of a beam to the centre of the nearest longitudinal bar shall be not greater than 100 mm.

For the purpose of (a) and (b) above, any bar having a diameter less than half the diameter of the largest bar in the section shall be ignored.



Notes:
1. Compressive reinforcement and stirrups have been omitted for clarity.
2. Skin reinforcement is not necessary above the level of the neutral axis, calculated assuming a cracked section.
3. Skin reinforcement is to be placed within the stirrups.

Figure 5.1 Crack Control Detailing Rules to AS 3600-1994

Clause 8.6.3 Crack control in the side face of beams (Clause 8.6, Crack Control of Beams)

For crack control in the side face of beams where the overall depth exceeds 750 mm, longitudinal reinforcement, consisting of Y12 bars at 200 mm centres, or Y16 bars at 300 mm centres, shall be placed in each side face (see Fig. 5.1(b)).

Clause 8.6.4 Crack control at openings and discontinuities (Clause 8.6, Crack Control of Beams)

Reinforcement shall be provided for crack control at openings and discontinuities in a beam.

5.3 AS 3600 – 2000 (Proposed Rules)

A simplified approach to design for crack control in Eurocode 2 – Clause 4.4.2.3 Control of Cracking without Direct Calculation, which avoids the calculation of crack widths, has been proposed for inclusion in AS 3600-2000, although with some modifications. The revised rules proposed for design for crack control of beams in flexure or tension in AS 3600-2000 are as follows.

Clause 2.4.4 Cracking (Clause 2.4, Design for Serviceability)

The cracking of beams or slabs under service conditions shall be controlled in accordance with the requirements of Clause 8.6 or 9.4 as appropriate.

Clause 8.1.4.1 General (Clause 8.1.4, Minimum Strength Requirements)

The ultimate strength in bending, M_{uo}, at critical sections shall not be less than $(M_{uo})_{min.}$ given by-

$$(M_{uo})_{min.} = 1.2\,[Z\,(f'_{cf} + P/A_g) + Pe] \qquad 5.3(1)$$

where –

Z = section modulus of the uncracked section, referred to the extreme fibre at which flexural cracking occurs;

f'_{cf} = characteristic flexural tensile strength of the concrete; and

e = the eccentricity of the prestressing force (P), measured from the centroidal axis of the uncracked section.

This requirement may be waived at some critical sections of an indeterminate member provided it can be demonstrated that this will not lead to sudden collapse of a span (see Fig. 5.2).

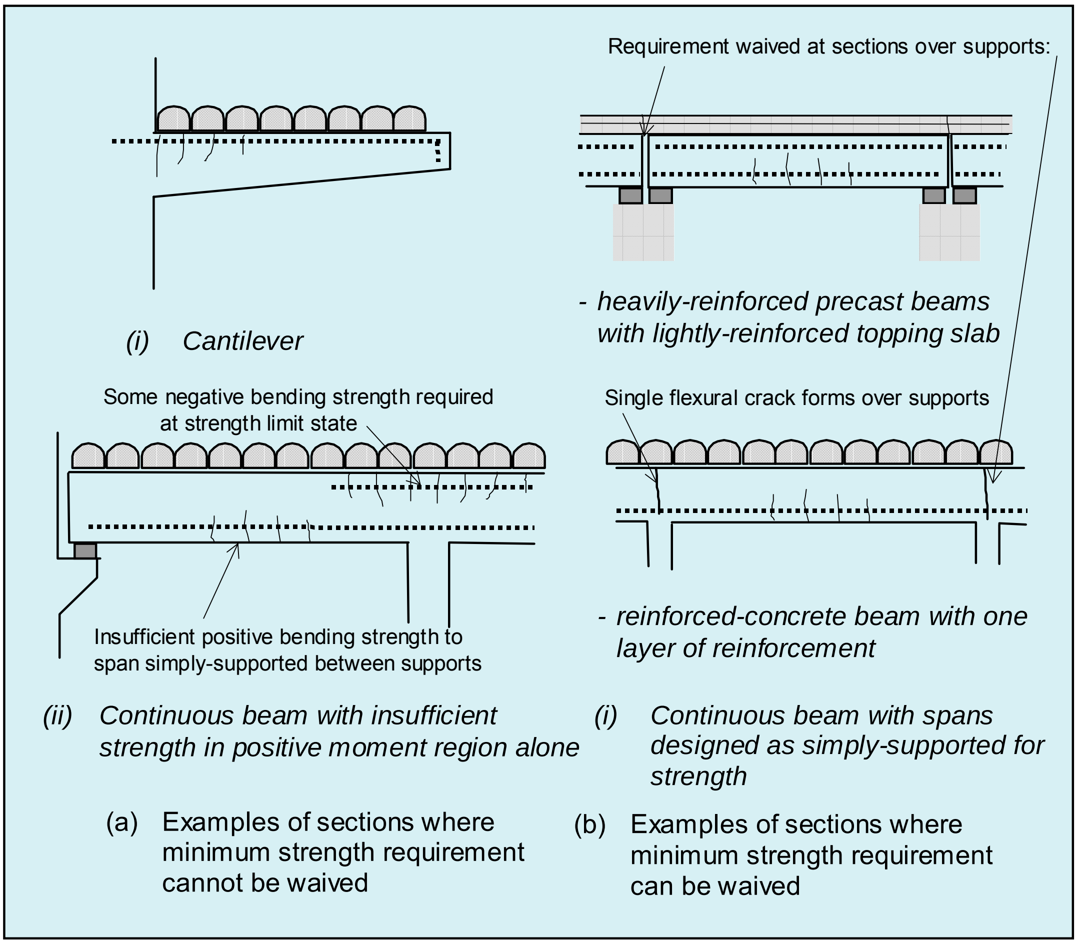


Figure 5.2 Sudden Collapse and the Minimum Strength Requirement

For prismatic, rectangular reinforced concrete sections, the requirement that $M_{uo} \geq (M_{uo})_{min.}$ shall be deemed to be satisfied for the direction of bending being considered if minimum tension reinforcement is provided such that –

$$A_{st}/bd \;\; \geq \;\; 0.22\,(D/d)^2\, f'_{cf}/f_{sy} \qquad\qquad 5.3(2)$$

For reinforced T-beams or L-beams where the web is in tension, b shall be taken as b_w.

Clause 8.1.7 Spacing of reinforcement and tendons (Clause 8.1, Strength of Beams in Bending)

The minimum clear distance between parallel bars (including bundled bars), ducts and tendons shall be such that the concrete can be properly placed and compacted in accordance with Clause 19.1.3 (Handling, Placing and Compacting of Concrete).[4] The maximum spacing of longitudinal reinforcement and tendons shall be determined in accordance with Clause 8.6 (Crack Control of Beams).

Clause 8.6.1 Crack control for tension and flexure in reinforced beams (Clause 8.6, Crack Control of Beams)

Cracking in reinforced beams subject to tension or flexure shall be deemed to be controlled if the appropriate requirements in items (a), (b) and (c), and either item (d) for beams primarily in tension or item (e) for beams primarily in flexure, are satisfied. In cases when the reinforcement has different yield strengths, in this clause the yield strength (f_{sy}) shall be taken as the lowest yield strength of any of the reinforcement.

For the purpose of this Clause, the resultant action is considered to be *primarily tension* when the whole of the section is in tension, or *primarily flexure* when the tensile stress distribution within the section prior to cracking is triangular with some part of the section in compression.

(a) The minimum area of reinforcement in the tensile zone ($A_{st.min}$) shall be:

$$A_{st.min} = 3\, k_s\, A_{ct}\, /f_s \qquad\qquad 5.3(3)$$

where –

k_s = a coefficient that takes into account the shape of the stress distribution within the section immediately prior to cracking, as well as the effect of non-uniform self-equilibrating stresses, and equals 0.8 for tension and 0.6 for flexure;

A_{ct} = the area of concrete in the tensile zone, being that part of the section in tension assuming the section is uncracked (see Figs 5.3 and 5.4 for case of flexure, noting that for tension it equals the whole of the concrete area); and

f_s = the maximum tensile stress permitted in the reinforcement immediately after formation of a crack, which shall be the lesser of the yield strength of the reinforcement (f_{sy}) and the maximum steel stress given in Table 8.6.1(A) of AS 3600 (see also Fig. 5.5) for the largest nominal diameter (d_b) of the bars in the section.

Note: This equation has been derived from Eq. 3.5.3(1) by putting f_t=3.0 MPa, and $k_s = k_3 \times k_4$, whereby $k_s = 0.8 \times 1.0$ for a state of tension, and $k_s = 1.0 \times 0.6$ for a state of flexure (see Section 3.6.1).

(b) The distance from the side or soffit of a beam to the centre of the nearest longitudinal bar does not exceed 100 mm. Bars with a diameter less than half the diameter of the largest bar in the section shall be ignored. The centre-to-centre spacing of bars near a tension face of the beam shall not exceed 300 mm.

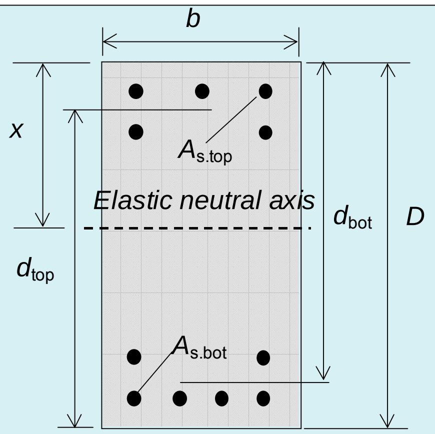
[4] Specific advice for the minimum clear distance can be found in Section 5.4.

(c) When direct loads are applied:

- the load combination for serviceability design with short-term effects shall be used in items (d) and (e) as appropriate, when calculating the steel stress, f_{scr}; and
- the steel stress assuming a cracked section, f_{scr}, shall also be calculated using the load combination for serviceability design with short-term effects, but with load factors of unity, and for this case shall not exceed a stress of $0.8f_{sy}$.

(d) For beams subject to tension, the steel stress (f_{scr}) calculated assuming the section is cracked does not exceed the maximum steel stress given in Table 8.6.1(A) for the largest nominal diameter (d_b) of the bars in the section.

(e) For beams subject to flexure, the steel stress (f_{scr}) calculated assuming the section is cracked does not exceed the maximum steel stress given in Table 8.6.1(A) for the largest nominal diameter (d_b) of the bars in the tensile zone. Alternatively, the steel stress does not exceed the maximum stress determined from Table 8.6.1(B) for the largest centre-to-centre spacing of adjacent parallel bars in the tensile zone (see Appendix E). Bars with a diameter less than half the diameter of the largest bar in the section shall be ignored when determining spacing. (See Figs 5.7, 5.8 and 5.9 for equations to calculate f_{scr}.)



(a) *Approximation ignoring presence of reinforcement*:

For both negative and positive bending –

$$A_{ct} = bD/2$$

(b) *Elastic analysis assuming uncracked section:*

For negative bending ($A_{s.top}$ in tension) –

$$A_{ct} = bx$$

For positive bending ($A_{s.bot}$ in tension) –

$$A_{ct} = b(D - x)$$

$x = \bar{k}D$ where -

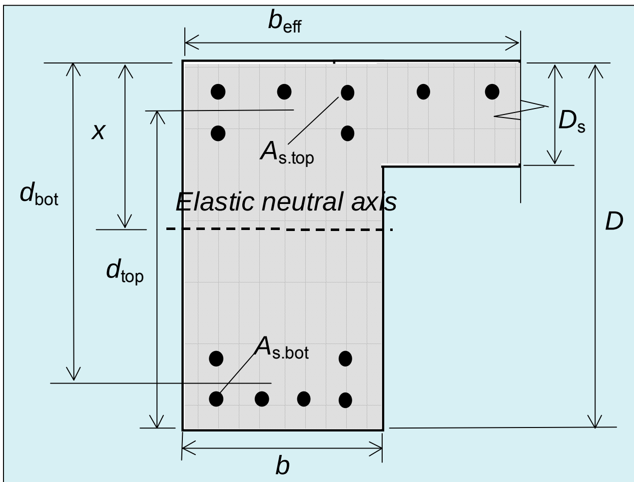
$\bar{k} = (0.5 + X)/(1 + Y); \bar{p} = A_{s.bot}/(bD); n = E_s/E_c$

$X = (n-1)\bar{p}(d_{bot}/D)[1 + A_{s.top}(D - d_{top})/(A_{s.bot}d_{bot})]$

$Y = (n-1)\bar{p}(1 + A_{s.top}/A_{s.bot})$

Refer to Eq. 7.2(2) to calculate I_{uncr} for a rectangular section.

Figure 5.3 Area of Concrete (A_{ct}) in Tensile Zone of a Rectangular Beam



(a) *Approximation ignoring presence of reinforcement*:

Use the equations below derived using elastic analysis, but put $A_{s.top}$ and $A_{s.bot}$ equal to zero.

(b) *Elastic analysis assuming uncracked section:*

For negative bending ($A_{s.top}$ in tension) –

$$A_{ct} = b_{eff}x \quad \text{if } x \le D_s$$
$$A_{ct} = b_{eff}D_s + b(x - D_s) \text{ if } x > D_s$$

For positive bending ($A_{s.bot}$ in tension) –

$$A_{ct} = b(D - x) \text{ if } x \ge D_s$$
$$A_{ct} = b(D - D_s) + b_{eff}(D_s - x) \quad \text{if } x < D_s$$

$x = \bar{k}D$ where-

$\bar{k} = (0.5 + \bar{X})/(1 + \bar{Y}); \bar{p} = A_{s.bot}/(bD)$

$\bar{X} = 0.5(D_s/D)^2(b_{eff}/b - 1) + X; n = E_s/E_c$

$\bar{Y} = (D_s/D)(b_{eff}/b - 1) + Y$

$X = (n-1)\bar{p}(d_{bot}/D)[1 + A_{s.top}(D - d_{top})/(A_{s.bot}d_{bot})]$

$Y = (n-1)\bar{p}(1 + A_{s.top}/A_{s.bot})$

Refer to Eq. 7.2(2) to calculate I_{uncr} for an L-beam or T-beam.

Figure 5.4 Area of Concrete (A_{ct}) in Tensile Zone of an L-Beam or T-Beam

Note: The designer should note that the equations in Fig. 5.4, which are based on the assumption of a horizontal neutral axis, only apply for an L-beam when the slab on one side is continuous and prevents lateral deflection of the beam.

TABLE 8.6.1(A) of AS 3600 - 2000
MAXIMUM STEEL STRESS FOR TENSION OR FLEXURE
(see Fig. 5.5)

Nominal bar diameter (d_b) (mm)	Maximum steel stress (MPa)
6	450
8	400
10	*360*
12	*330*
16	*280*
20	*240*
24	*210*
28	*185*
32	160
36	140
40	120

NOTES:
1. Use of bar diameters less than or equal to 28 mm, shown in italics, is preferred in this booklet.
2. Sizes 6 and 8 mm are not available as separate bars.
3. The values in the table can be accurately calculated using the equation (also see dashed curve in Fig. 5.5): Maximum steel stress = $-173\log_e(d_b) + 760$ MPa

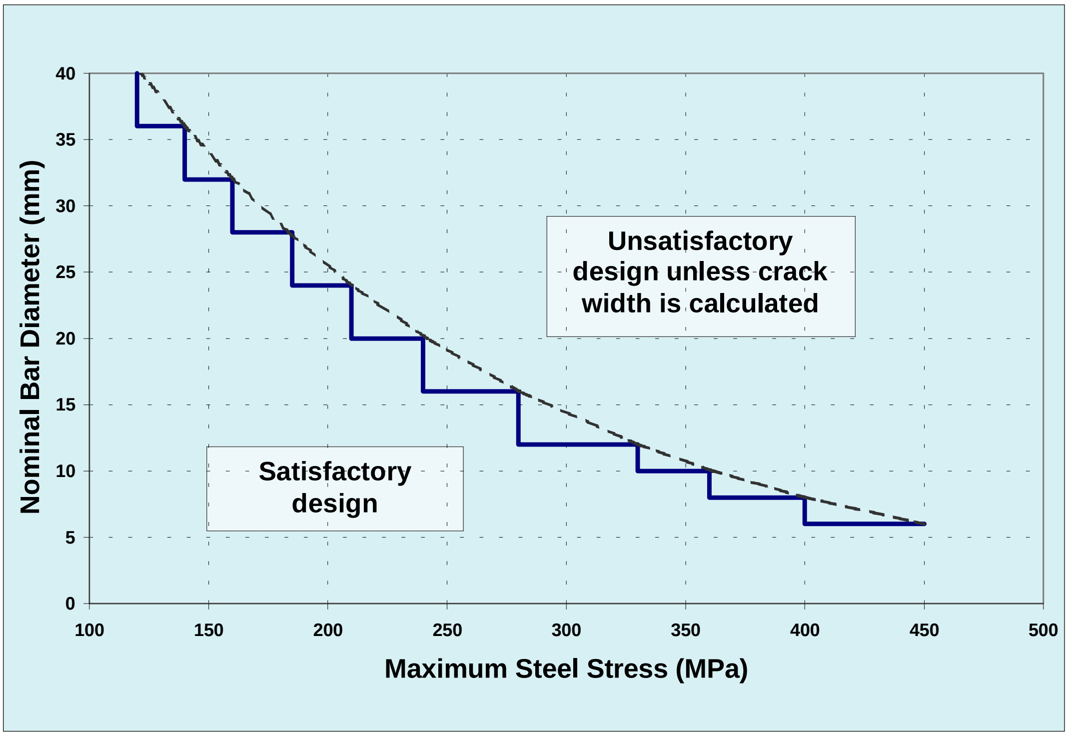


Figure 5.5 Maximum Steel Stress as a Function of Nominal Bar Diameter (Tension or Flexure) – Table 8.6.1(A) of AS 3600

TABLE 8.6.1(B) of AS 3600 - 2000
MAXIMUM STEEL STRESS FOR FLEXURE
(see Fig. 5.6)

Centre-to-centre spacing (mm)	Maximum steel stress (MPa)
50	360
100	320
150	280
200	240
250	200
300	160

NOTE: Linear interpolation may be used with the equation:

Maximum steel stress = – 0.8 × centre-to-centre spacing + 400 MPa

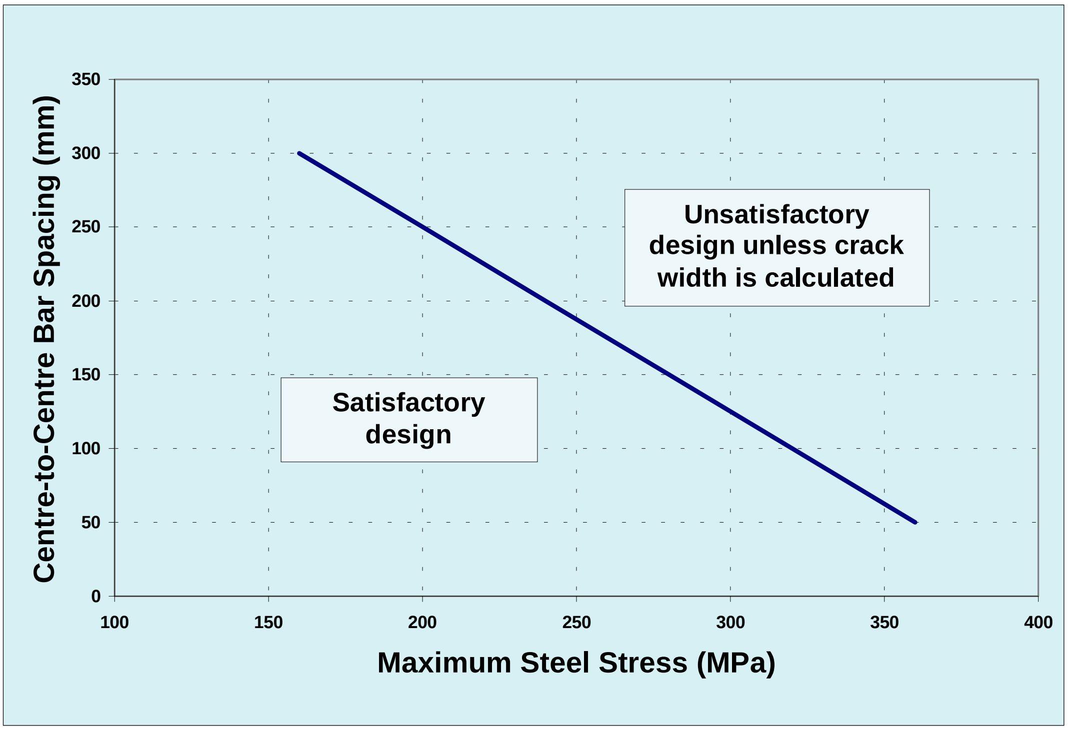
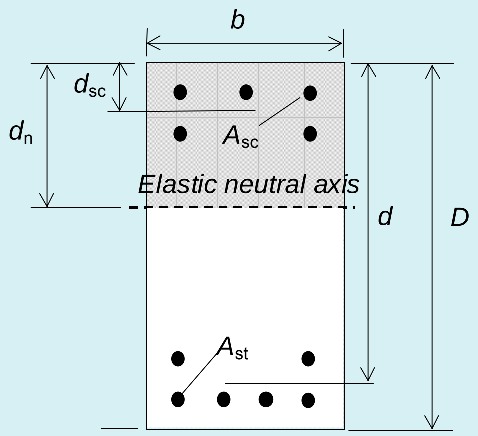


Figure 5.6 Centre-to-Centre Bar Spacing as a Function of Maximum Steel Stress (Flexure) – Table 8.6.1(B) of AS 3600



$$f_{scr} = nM_s^*(d - d_n)/I_{cr} \quad \text{or} \quad f_{scr} = M_s^*/(A_{st}z)$$

where -

$$d_n = kd$$

$$k = -Y + \sqrt{Y^2 + 2X}\,;\, p = A_{st}/(bd);\, n = E_s/E_c$$

$$X = np[1 + (n-1)A_{sc}d_{sc}/(nA_{st}d)]$$

$$Y = np[1 + (n-1)A_{sc}/(nA_{st})]$$

and

$$I_{cr} = \kappa bd^3/12$$

$$\kappa = 4k^3 + 12np(1-k)^2 + 12(n-1)p(A_{sc}/A_{st})(k - d_{sc}/d)^2$$

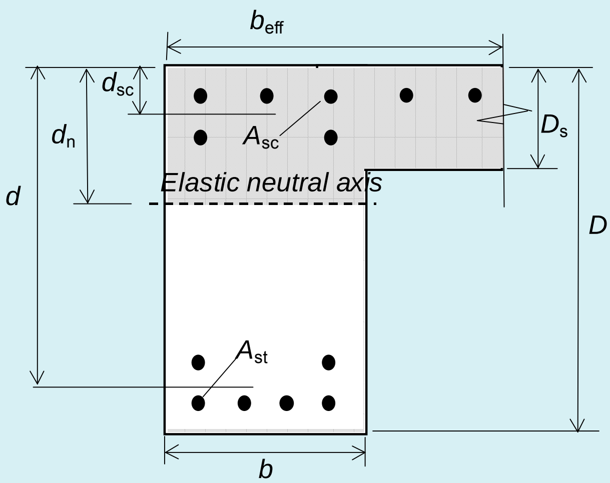
and

$$z = \frac{(d - d_{sc})2A_{sc}(n-1)(d_n - d_{sc}) + (d - d_n/3)bd_n^2}{2A_{sc}(n-1)(d_n - d_{sc}) + bd_n^2}$$

Notes:

1. The steel tensile stress, f_{scr}, is calculated at the depth of the centroid of the tension reinforcement, which is considered satisfactory for close layers.
2. The case of $d_n \le d_{sc}$ is not considered practical.

Figure 5.7 Calculation of f_{scr} for a Rectangular Beam – Positive or Negative Bending



$f_{scr} = nM^*_s(d - d_n)/I_{cr}$

where -

$d_n = kd$

$k = -\overline{Y} + \sqrt{\overline{Y}^2 + \overline{X}}\,;p = A_{st}/(bd);n = E_s/E_c$

$\overline{X} = (D_s/d)^2(b_{eff}/b - 1) + 2X$

$\overline{Y} = (D_s/d)(b_{eff}/b - 1) + Y$

$X = np[1 + (n-1)A_{sc}d_{sc}/(nA_{st}d)]$

$Y = np[1 + (n-1)A_{sc}/(nA_{st})]$

and

$I_{cr} = \kappa bd^3/12$

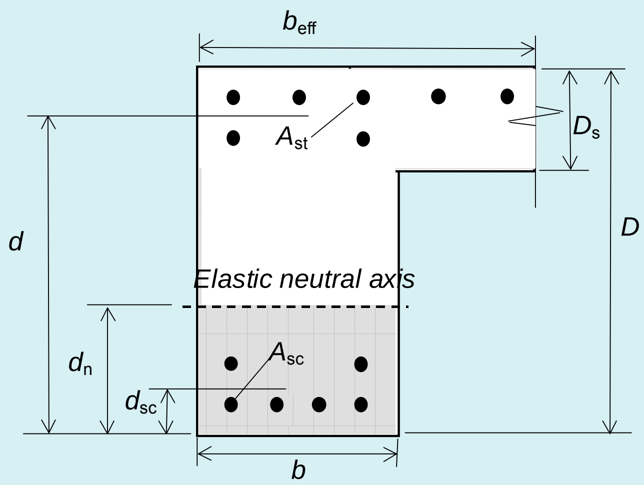
$$\kappa = 4\left[\frac{b_{eff}}{b}k^3 - \left(\frac{b_{eff}}{b} - 1\right)\left(k - \frac{D_s}{d}\right)^3\right] + 12np(1-k)^2 + 12(n-1)p(A_{sc}/A_{st})(k - d_{sc}/d)^2$$

Notes:

1. If $d_n < D_s$, then perform calculations again treating section as rectangular, replacing b with b_{eff} in equations in Fig. 5.7.
2. The alternative equation for calculating f_{scr}, as shown in Fig. 5.7, has been omitted since the equation for z is too complex to be practical use.
3. The steel tensile stress, f_{scr}, is calculated at the depth of the centroid of the tension reinforcement, which is considered satisfactory for close layers.
4. The case of $d_n \leq d_{sc}$ is not considered practical.

Figure 5.8 Calculation of f_{scr} for an L-Beam or T-Beam – Positive Bending

Note: The designer should note that the equations in Fig. 5.8, which are based on the assumption of a horizontal neutral axis, only apply for an L-beam when the slab on one side is continuous and prevents lateral deflection of the beam.



$$f_{scr} = nM_s^*(d - d_n)/I_{cr} \quad \text{or} \quad f_{scr} = M_s^*/(A_{st}z)$$

where -

$$d_n = kd$$

$$k = -Y + \sqrt{Y^2 + 2X}\ ; p = A_{st}/(bd); n = E_s/E_c$$

$$X = np[1 + (n-1)A_{sc}d_{sc}/(nA_{st}d)]$$

$$Y = np[1 + (n-1)A_{sc}/(nA_{st})]$$

and

$$I_{cr} = \kappa bd^3/12$$

$$\kappa = 4k^3 + 12np(1-k)^2 + 12(n-1)p(A_{sc}/A_{st})(k - d_{sc}/d)^2$$

and

$$z = \frac{(d - d_{sc})2A_{sc}(n-1)(d_n - d_{sc}) + (d - d_n/3)bd_n^2}{2A_{sc}(n-1)(d_n - d_{sc}) + bd_n^2}$$

Notes:

1. These equations are exactly the same as those in Fig. 5.7 for a rectangular section with a compressive flange of width b.
2. The steel tensile stress, f_{scr}, is calculated at the depth of the centroid of the tension reinforcement, which is considered satisfactory for close layers.
3. The cases of $d_n \le d_{sc}$ or $d_n > (D - D_s)$ are not considered practical.

Figure 5.9 Calculation of f_{scr} for an L-Beam or T-Beam – Negative Bending

Note: The designer should note that the equations in Fig. 5.9, which are based on the assumption of a horizontal neutral axis, only apply for an L-beam when the slab on one side is continuous and prevents lateral deflection of the beam.

Clause 8.6.3 Crack control in the side face of beams (Clause 8.6, Crack Control of Beams)

For crack control in the side face of beams where the overall depth exceeds 750 mm, longitudinal reinforcement, consisting of 12 mm diameter bars at 200 mm centres, or 16 mm diameter bars at 300 mm centres, shall be placed in each side face (see Fig. 5.1(b)).

Clause 8.6.4 Crack control at openings and discontinuities (Clause 8.6, Crack Control of Beams)

Reinforcement shall be provided for crack control at openings and discontinuities in a beam.

5.4 Additional Design Rules

The following design rules, which are additional to those in AS 3600-1994 or those proposed to date for AS 3600-2000, are required to design and detail beams for crack control.

Exposure Classifications Requiring Crack Control

The requirements of AS 3600-1994 for crack control shall apply to all exposure classifications. For AS 3600-2000, it is further proposed that the requirements shall only apply to Exposure Classification A1 when crack control is required for aesthetic reasons.

Minimum Spacing of Reinforcing Bars

A minimum clear distance between parallel bars is not specified in AS 3600, but is needed to allow the concrete to flow into place. It is recommended that the minimum clear distance is restricted to the larger of 1.5 times the maximum nominal size of aggregate (normally max. aggregate size is 20 mm) and the diameter of the largest reinforcing bar [1]. It is also worth noting that the centre of a corner bar enclosed in a stirrup of diameter d_s cannot be located closer than the greater of R_s and $0.5d_b$ to the inside edge of the vertical leg of the stirrup, where R_s is the corner radius of the stirrup. In accordance with Clause 19.2.3.2, $2R_s$=$4d_s$ for 400 MPa and 500 MPa fitments. This is important when determining the minimum width of a beam.

Maximum Bar Diameter

A maximum bar diameter of 28 mm is recommended for the main tension reinforcement in beams when crack control is important [1]. Nevertheless, bars of larger diameter can be used in accordance with AS 3600-2000, but their cross-sectional area in tensile regions is likely to be governed by the maximum serviceability stress, f_s, rather than yield strength, f_{sy}, unless they are closely spaced. Bundled bars can be treated as in Eurocode 2 (see Section 3.7.2).

Monolithic Construction

The possibility of partial fixity at simple supports should be considered (see Section 3.7.4). The normal design provisions of AS 3600-1994 should be followed in this regard; e.g. Clause 7.2.2(c) for beams designed using the Simplified Method.

6. COMPUTER SOFTWARE

6.1 General

A computer program described in the next Sub-section has been written to assist with the design of reinforced-concrete beams for crack control. The designs are in accordance with the rules in Section 5.3, which have been proposed for inclusion in AS 3600-2000.

The simple nature of the crack control design rules in AS 3600-1994 has meant that crack control has not formed a major part of the overall design process of a reinforced-concrete structure or member. Satisfactory crack control in beams has appeared to be assured by simply spacing main bars at 200 mm centres or less, keeping them within 100 mm of the tension face. Many other design factors that can significantly increase the likelihood of cracking have been ignored, e.g. the degree of moment redistribution assumed at the strength limit state (see Section 4.1).

Following on from Section 4.1, a major objective of the software is, whenever possible, to find satisfactory solutions for crack control which provide the full benefit of the increase in steel strength to 500 MPa.

It follows that the software can be usefully used to check designs that are otherwise complete and ready for final detailing. Some designs might have been performed assuming 400 MPa steel will be used, leaving crack control a major design issue to address before converting to 500PLUS Rebar. As necessary, critical regions must be checked for crack control, and the diameter and spacing of the main reinforcing bars selected accordingly. It is assumed that the dimensions of the concrete beam are known prior to running the software.

6.2 Computer Program 500PLUS-BCC™

The design approach presented in Figs 4.2 to 4.5 in Section 4.3 explains the steps a designer will normally follow when performing detailed calculations for crack control. Of course, a major difficulty with performing these calculations manually is that a number of iterations may have to be performed before finding a satisfactory solution. At the very least, it would seem prudent for a designer to program the equations in Figs 5.3, 5.4, etc. in order to reduce the amount of manual calculation. However, the success of designing using this approach will depend somewhat on the suitability of the initial estimate of bar diameter. The designer should also check that the solution is the most efficient and if possible minimise steel area.

Computer program 500PLUS-BCC allows quite a different approach to be taken during design. No manual calculations are required once the input data has been determined. Solutions that allow the full benefit to be gained from using 500PLUS Rebar are clearly identified as “strength governs”. These are the solutions for which crack control design does not govern the area of tension reinforcement. Otherwise, the solutions are identified as “serviceability governs”, and they may not be the most suitable.

It is not necessary to specify the actual areas of reinforcement in the beam. If these are known beforehand, then they can be compared and adjusted as necessary with the results from the crack control analysis. However, if the beam is being designed for a flexural state, and the area of compressive reinforcement is known, then this should be input for each section since this is taken into account by the computer program.

The program is structured such that the user first defines the cross-section details. The beam is assumed to have a prismatic section and to be continuous with both sagging (positive) and hogging (negative) moment regions. The maximum design action effects for both of these regions are then defined, except that if the designer has determined that a state of tension exists, then only one of these regions need be considered during design. Sagging bending results in tension in the bottom face under the action of the bending moment alone.

The steps to follow when using the program to design the critical sections of a reinforced-concrete beam are briefly explained as follows (see Figs 6.1 to 6.6).

1. The cross-section details must be input first (see Fig. 6.1). A section can have either a rectangular, T or L shape. The span must be known to be able to compute the effective width of the flange for T- or L-beams in accordance with Clause 8.8.2 of AS 3600-1994. The main dimensions that define the cross-section, and the covers to the main reinforcement, are required (see Section 5.4, Minimum Spacing of Reinforcing Bars). The overall depth of the beam, D, cannot be assigned less than 300 mm. The side cover must not be too large to invalidate the limit of 100 mm to the centre of edge bars. If the beam is rectangular then simply assign $b_{eff}=b$. The reinforcement in the bottom of the beam may be placed in two layers, if there is insufficient space for one layer. The gap (clear distance) between these layers must be defined using the Options feature (see Fig. 6.2). The minimum horizontal gap is also defined using this feature. Only one layer is ever placed in the top face. If the beam is rectangular, the outer top bars are checked for cover like the bottom outer bars. Otherwise, it is assumed that the top bars are equi-spaced across the effective width of the beam with an edge distance equal to half the bar spacing. The compressive strength of the concrete, f'_c (20 to 50 MPa) and the yield strength of the steel, f_{sy} (400 or 500 MPa) must also be specified.

2. The design action effects for both the strength and serviceability limit states must have been determined separately from analysis (see Table 4.3(1)). For sections deemed to be in a state of flexure, it is also necessary to input the bending moment corresponding to the serviceability overload condition with full live load applied to the beam, i.e. G+Q loading. The critical sections for the sagging and hogging moment regions are considered separately (see Figs 6.3 and 6.5, respectively).

3. Once the data has been input, the program will automatically perform the necessary calculations and immediately display the final results. Firstly, it will indicate whether the section is deemed to be in flexure or tension (see Fig. 3.3). This will be printed directly beneath where the design action effects are input. The main output consists of a table of feasible solutions.

4. The table of solutions may extend over the full range of 400 MPa and 500PLUS Rebar sizes produced by OneSteel Reinforcing, viz. 10 (500PLUS only), 12, 16, 20, 24, 28, 32, 36 and 40 mm. The smallest and largest bar diameters of 10 and 40 mm are new additions to the bar range. The 10 mm bar is particularly suited to controlling cracking in small beams supporting relatively high serviceability loads.

   A feature of the table is that it contains every feasible solution that satisfies design for strength, and design for crack control in accordance with all of the requirements of Section 5.3. It is up to the designer to choose the design that best suits the situation. A dash is placed where no feasible solution exists. This occurs for the smaller diameter bars when there is insufficient space between the bars. Using two layers of reinforcement, particularly in the bottom face of T- and L-beams, will alleviate this problem. Solutions shown in green indicate two layers have been used (see Fig. 6.3). A minimum of two bars is placed in the top layer of the bottom steel. It is up to the designer to determine the exact disposition of bars when multiple layers are specified in the output. The designer should first calculate the number of bars in the bottom layer knowing the covers, bar diameter and spacing, and then assume that the remaining bars are in the top layer.

   Solutions shown in a lightly-shaded area (see Fig. 6.5) indicate that the crack control design rules governed the area of tension steel. In such a case, the yield strength of the reinforcing bars cannot be fully utilised at the strength limit state. Although the solution is feasible, it may not be the most economical since a larger steel area is required.

5. The calculations behind each solution in a table can be examined by clicking on the "Prop.(erties)" button to the right of the table. This also applies to non-feasible situations where there is a dash. Sample results are shown in Fig. 6.4 for one of the bar diameters for which there is a feasible solution in Fig. 6.3, and similarly in Figs 6.5 and 6.6.

6. A screen can be printed directly by clicking on the "Print" button.

7. The program can be terminated by clicking on the "Close" button.

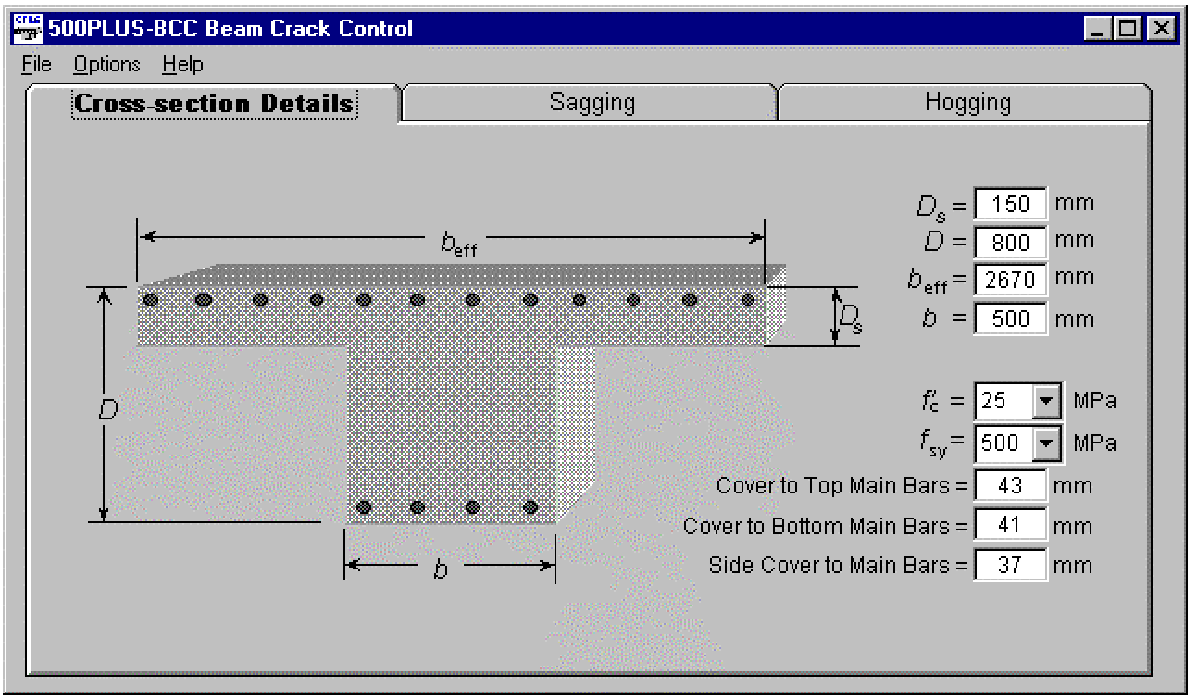


Figure 6.1 Program 500PLUS-BCC™ – Cross-section Details

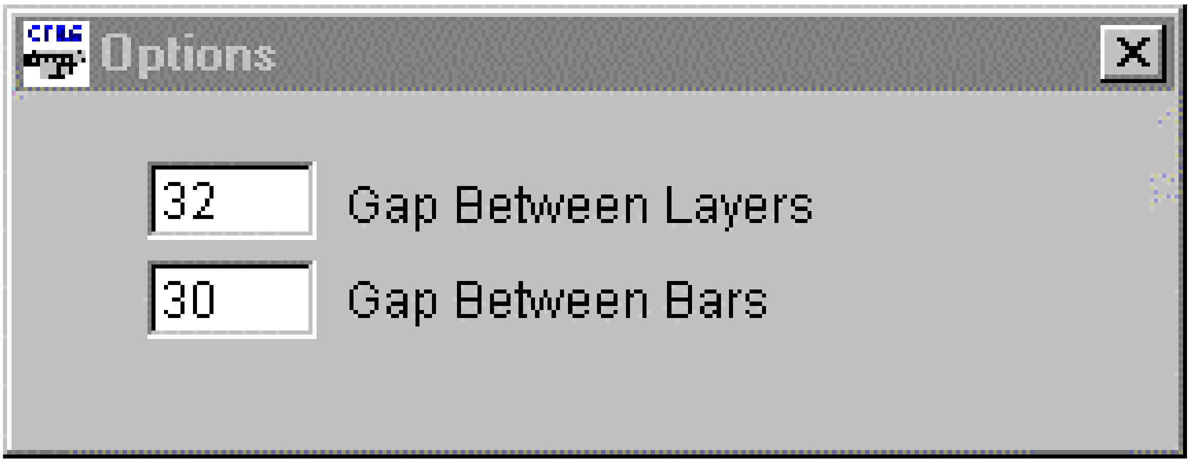


Figure 6.2 Program 500PLUS-BCC™ – Cross-section Detail Options

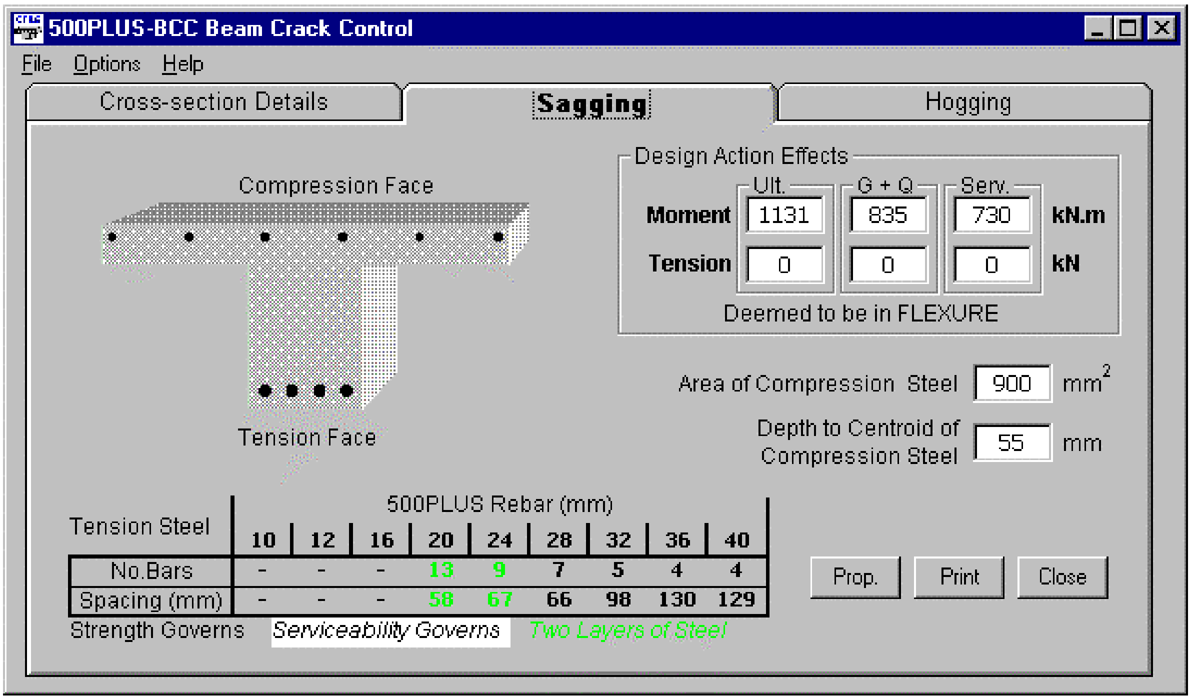


Figure 6.3 Program 500PLUS-BCC™ – Sagging Bending

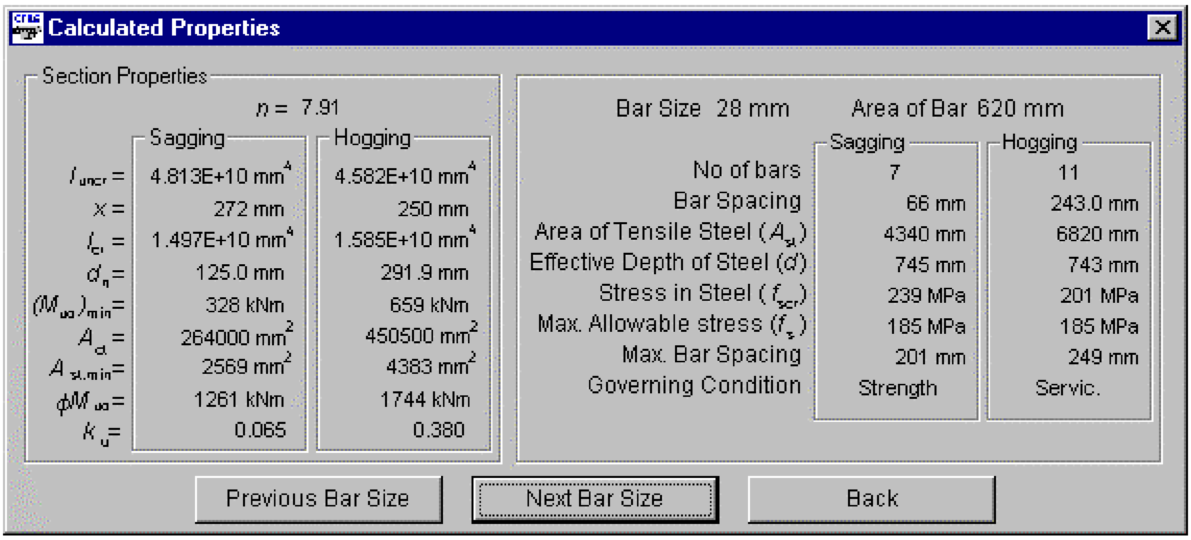


Figure 6.4 Program 500PLUS-BCC™ – Sagging Bending Calculated Properties (d_b=28 mm)

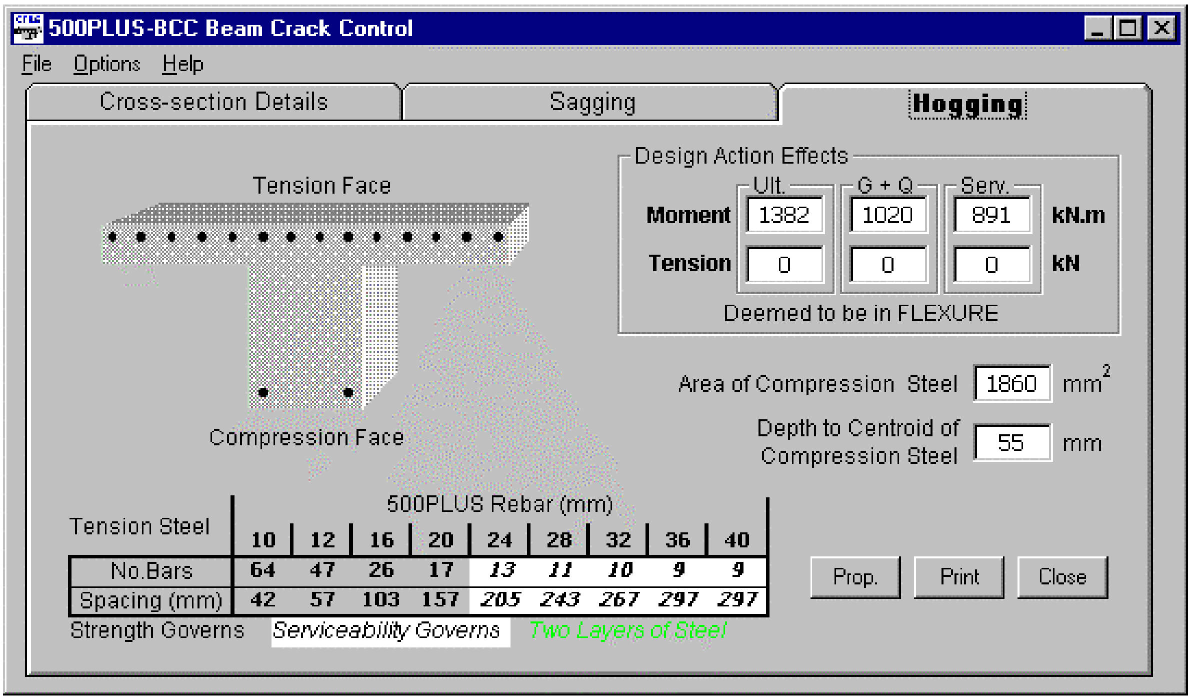


Figure 6.5 Program 500PLUS-BCC™ – Hogging Bending

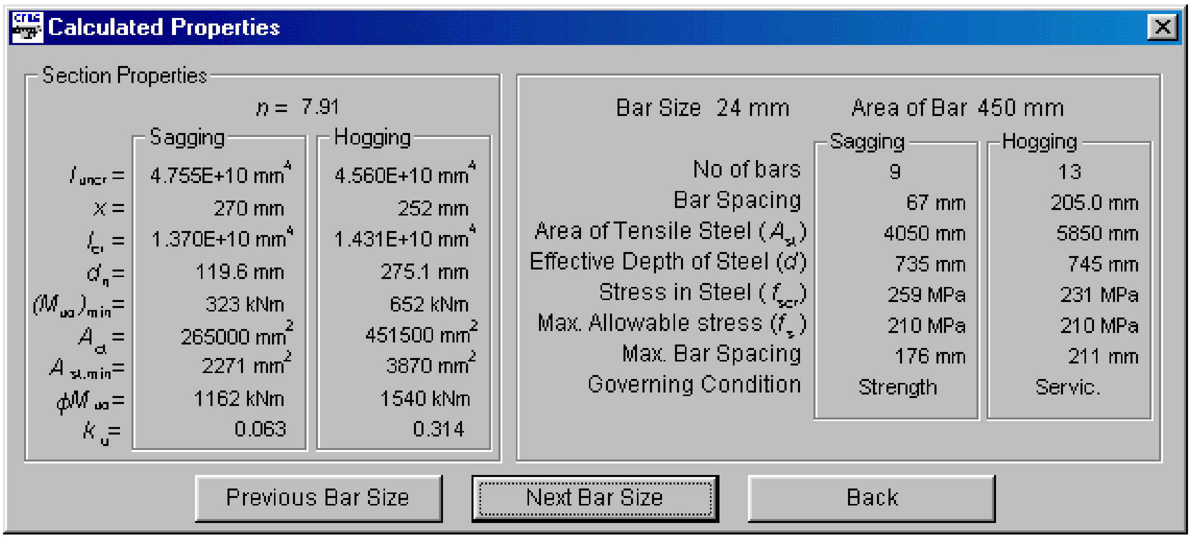


Figure 6.6 Program 500PLUS-BCC™ – Hogging Bending Calculated Properties (d_b=24 mm)

7. WORKED EXAMPLES

7.1 General

Several worked examples are used to illustrate to design engineers the way they should use the new rules proposed for inclusion in AS 3600-2000 to design reinforced-concrete beams for crack control. Some typical design situations are examined. The opportunity is also taken to show the benefits that can be obtained using the higher strength 500 MPa reinforcing steels, viz. 500PLUS Rebar, in beams. This can lead to a significant reduction in steel areas and less congestion, resulting in obvious economic benefits when crack control does not govern design.

The reader can also refer to other documents for design examples that illustrate the use of the crack control design provisions in Eurocode 2 [10,11,19]. However, the examples given in these documents for reinforced-concrete beams are very limited.

7.2 Example 1 - Two-Span Continuous T- Beam Originally Designed to AS 3600-1994 Using 400 MPa Bars, Redesigned to AS 3600-2000 Using 500PLUS® Rebar

This example is used to illustrate how a designer might review a design conducted in accordance with AS 3600-1994 with a view to converting the 400 MPa main bars to 500PLUS Rebar. Crack control is a critical design issue to consider before the cross-sectional area of the main reinforcement can be reduced and the benefits (arising from less steel) gained from moving to the higher strength grade.

Warner et al. [1] have designed a two-span continuous reinforced-concrete beam and detailed the critical sections as shown in Fig. 7.1 (Example 16.2 therein). All the information necessary to design these sections for crack control is included in the figure. The design bending moments at the strength limit state were calculated by Warner et al. using the Simplified Method in Section 7.2 of AS 3600-1994. Therefore, the design bending moment for serviceability, M_s^*, in both the peak hogging and sagging moment regions has been estimated using the following formulae, which derives directly from Eq. 4.1(2), noting that η=0 for the Simplified Method:

$$M_s^* = M^* \frac{G + \psi_s Q}{1.25G + 1.5Q} \qquad 7.2(1)$$

It has been assumed that ψ_s=0.7 in Eq. 7.2(1). Also, $M_{s.1}^*$ has been calculated by putting ψ_s=1.0 in the same equation.

The example comprises two parts, viz.:

Part 1 – The original design using 400 MPa bars shown in Fig. 7.1 is checked for crack control. The design is found to be marginal, and improvements are recommended.

Part 2 – The design is converted to 500PLUS Rebar. Advantage is taken of the reduced cross-sectional area of bars required for bending strength in both the negative and positive moment regions. It is shown that it is possible to increase the effective depths, leading to even greater reductions in the original quantity of steel required. Fewer bars can be used in the positive moment region where the bars are closely spaced, and the stress in the tension steel under serviceability loads can be high.

Part 1 – Original design using 400 MPa bars (see Fig. 7.1)

As a formality in this case, the minimum strength requirement shall be checked according to Fig. 4.2. Then, the flowcharts in Figs 4.3, 4.4 and 4.5 are followed as appropriate. With the serviceability design tensile force, T_s^*, assumed to equal zero, it is clear that there is a state of flexure throughout the beam. The presence of the heavier-than-required skin reinforcement nominated by Warner et al. will be conservatively ignored during the calculations.

Peak negative moment section

The uncracked section properties are calculated as follows. Firstly, from Fig. 5.4:

$$\begin{aligned}\bar{p} &= A_{s.bot}/(bD)\\ &= 1860/(500\times 800)\\ &= 0.00465\end{aligned}$$

$$\begin{aligned}n &= E_s/E_c\\ &= 200000/(0.043\rho_c^{1.5}\sqrt{f_{cm}}) \qquad \text{but will use } f_{cm} = f_c'\\ &= 200000/(0.043\times 2400^{1.5}\sqrt{25})\\ &= 7.9\end{aligned}$$

Note: steel stress calculations are relatively insensitive to the value of n.

d_{top} = 710 mm

d_{bot} = 750 mm

$$\begin{aligned}X &= (n-1)\bar{p}(d_{bot}/D)[1+A_{s.top}(D-d_{top})/(A_{s.bot}d_{bot})]\\ &= (7.9-1)\times 0.00465\times(750/800)\times[1+7440\times(800-710)/(1860\times 750)]\\ &= 0.044518\end{aligned}$$

$$\begin{aligned}Y &= (n-1)\bar{p}(1+A_{s.top}/A_{s.bot})\\ &= (7.9-1)\times 0.00465\times(1+7440/1860)\\ &= 0.160425\end{aligned}$$

$$\begin{aligned}\bar{X} &= 0.5(D_s/D)^2(b_{eff}/b - 1)+X\\ &= 0.5\times(150/800)^2\times(2670/500-1)+0.044518\\ &= 0.120807\end{aligned}$$

$$\begin{aligned}\bar{Y} &= (D_s/D)(b_{eff}/b - 1)+Y\\ &= (150/800)\times(2670/500-1)+0.160425\\ &= 0.974175\end{aligned}$$

$$\begin{aligned}\bar{k} &= (0.5+\bar{X})(1+\bar{Y})\\ &= (0.5+0.120807)/(1+0.974175)\\ &= 0.314464\end{aligned}$$

$$\begin{aligned}x &= \bar{k}D\\ &= 0.31446\times 800\\ &= 252\text{ mm}\end{aligned}$$

A further equation to that given in Fig. 5.4 is for the second moment of area of the uncracked section, viz:

$$I_{uncr} = \kappa b D^3/12 \qquad 7.2(2)$$

where –

$$\kappa = 1+(\frac{b_{eff}}{b}-1)(\frac{D_s}{D})^3+12(0.5-\bar{k})^2+12(\frac{b_{eff}}{b}-1)(\frac{D_s}{D})(\bar{k}-0.5\frac{D_s}{D})^2$$
$$+12(n-1)\bar{p}\left[\left(\frac{d_{bot}}{D}-\bar{k}\right)^2+\frac{A_{s.top}}{A_{s.bot}}\left\{\bar{k}-\frac{(D-d_{top})}{D}\right\}^2\right]$$

It follows that:

$$\kappa = 1 + (\frac{2670}{500} - 1)(\frac{150}{800})^3 + 12\times(0.5 - 0.314464)^2 + 12\times(\frac{2670}{500} - 1)(\frac{150}{800})(0.314464 - 0.5\times\frac{150}{800})^2$$

$$+12\times(7.9-1)\times0.00465\times\left[\left(\frac{750}{800} - 0.314464\right)^2 + \frac{7440}{1860}\left\{0.314464 - \frac{(800-710)}{800}\right\}^2\right]$$

$$= 2.130$$

Thus,

$$\begin{aligned} I_{\text{uncr}} &= \kappa b D^3/12 \\ &= 2.130\times500\times800^3/12 \\ &= 0.04543\times10^{12}\ \text{mm}^4 \end{aligned}$$

It follows from Eq. 5.3(1), noting that $Z = I_{\text{uncr}}/x$, that:

$$\begin{aligned} (M_{\text{uo}})_{\min} &= 1.2 Z f_{cf}^{'} \\ &= 1.2\times(0.04543\times10^{12}/252)\times0.6\sqrt{25}/10^6 \\ &= 649\ \text{kNm} \end{aligned}$$

According to Warner et al., the nominal negative moment capacity M_{uo}=1478/ϕ=1478/0.8=1847 kNm which is greater than $(M_{\text{uo}})_{\min}$. Therefore, the minimum strength requirement is clearly satisfied.

Further from Fig. 5.4, noting that $x > D_s$, it can be written that the area of concrete in tension immediately prior to cracking, A_{ct}, equals:

$$\begin{aligned} A_{\text{ct}} &= b_{eff} D_s + b(x - D_s) \\ &= 2670\times150 + 500\times(252-150) \\ &= 451500\ \text{mm}^2 \end{aligned}$$

The minimum area of tension reinforcement, $A_{\text{st.min}}$, can now be calculated using Eq. 5.3(3). In this equation, k_s=0.6 for flexure, A_{ct}=451500 mm^2 and f_s=185 MPa (from Table 8.6.1(A) of AS 3600-2000 for Y28 bars), whereby:

$$\begin{aligned} A_{\text{st.min}} &= 3 k_s A_{\text{ct}}/f_s \\ &= 3\times0.6\times451500/185 \\ &= 4393\ \text{mm}^2 \end{aligned}$$

Since the actual area of tension steel at the peak negative moment section, A_{st}=7440 mm^2, the minimum reinforcement requirement for crack control is clearly satisfied.

The next step is to calculate the stresses in the tension reinforcement after cracking, f_{scr}, under the action of M_s^* and $M_{s.1}^*$. For this purpose, Fig. 5.9 will be followed:

$$\begin{aligned} p &= A_{\text{st}}/(bd) \\ &= 7440/(500\times710) \\ &= 0.02096 \\ X &= np[1 + (n-1)A_{\text{sc}} d_{\text{sc}}/(n A_{\text{st}} d)] \\ &= 7.9\times0.02096\times[1 + (7.9-1)\times1860\times50/(7.9\times7440\times710)] \\ &= 0.1681 \end{aligned}$$

$$\begin{aligned}Y &= np[1+(n-1)A_{sc}/(nA_{st})]\\ &= 7.9\times0.02096\times[1+(7.9-1)\times1860/(7.9\times7440)]\\ &= 0.2017\end{aligned}$$

$$\begin{aligned}k &= -Y+\sqrt{Y^2+2X}\\ &= -0.2017+\sqrt{0.2017^2+2\times0.1681}\\ &= 0.4122\end{aligned}$$

$$\begin{aligned}d_n &= kd\\ &= 0.4122\times710\\ &= 293\text{ mm}\end{aligned}$$

Note: because d_n<(D - D_s) (=650 mm), use of the equations in Fig. 5.9 is valid.

$$\kappa = 4k^3+12np(1-k)^2+12(n-1)p\frac{A_{sc}}{A_{st}}(k-\frac{d_{sc}}{d})^2$$

$$=4\times0.4122^3+12\times7.9\times0.02096\times(1-0.4122)^2+12\times(7.9-1)\times0.02096\times\frac{1860}{7440}\times(0.4122-\frac{50}{710})^2$$

$$=1.0173$$

$$\begin{aligned}I_{cr} &= \kappa bd^3/12\\ &= 1.0173\times500\times710^3/12\\ &= 0.01517\times10^{12}\text{ mm}^4\end{aligned}$$

$$\begin{aligned}f_{scr.1} &= nM^*_{s.1}(d-d_n)/I_{cr}\\ &= 7.9\times1020\times10^6\times(710-293)/(0.01517\times10^{12})\\ &= 222\text{ MPa}\end{aligned}$$

$$\begin{aligned}f_{scr} &= nM^*_s(d-d_n)/I_{cr}\\ &= 7.9\times891\times10^6\times(710-293)/(0.01517\times10^{12})\\ &= 194\text{ MPa}\end{aligned}$$

It follows that because $f_{scr.1}$<0.8f_{sy} (=320 MPa), yielding the reinforcement under service loads is not possible, and the design is satisfactory in this respect.

However, f_{scr}>185 MPa, which is the maximum allowable stress from Table 8.6.1(A) of AS 3600-2000 for Y28 bars. Therefore, crack control in the top face of the beam is unsatisfactory. In accordance with Clause 8.6.1(e) of AS 3600-2000, the problem can be overcome by reducing the spacing of the bars. This must be done across the whole of the effective width of the beam, and in accordance with Table 8.6.1(B) the maximum acceptable centre-to-centre spacing equals 250 mm. This can be achieved by several ways, the most logical being simply to move the two Y28 bars in the lower layer into the top layer, placing four rather than three main bars in each flange outstand.

The appropriate positioning of these top tension bars in the effective width was not addressed by Warner et al. in their example. They seemed to favour concentrating the bars within the vicinity of the beam web. The calculations above show that four Y28 bars should be placed outside the stirrups on each side of the rectangular beam, leaving four Y28 bars to be contained by the stirrups in the top face. This improves crack control, and also means that the value of effective depth d (=710 mm) assumed in the calculations has been underestimated. This will be highlighted in the improved design performed for the negative moment section.

This completes the check of the peak negative moment section with the original design using 400 MPa bars.

Peak positive moment section

The uncracked section properties are calculated as follows. Firstly, from Fig. 5.4:

$$\begin{aligned}\bar{p} &= A_{s.bot}/(bD)\\ &= 6200/(500\times 800)\\ &= 0.0155\end{aligned}$$

d_{top} = 750 mm

d_{bot} = 710 mm

$$\begin{aligned}X &= (n-1)\bar{p}(d_{bot}/D)[1+A_{s.top}(D-d_{top})/(A_{s.bot}d_{bot})]\\ &= (7.9-1)\times 0.0155\times(710/800)\times[1+1240\times(800-750)/(6200\times 710)]\\ &= 0.09626\end{aligned}$$

$$\begin{aligned}Y &= (n-1)\bar{p}(1+A_{s.top}/A_{s.bot})\\ &= (7.9-1)\times 0.0155\times(1+1240/6200)\\ &= 0.12834\end{aligned}$$

$$\begin{aligned}\bar{X} &= 0.5(D_s/D)^2(b_{eff}/b\text{ - }1)+X\\ &= 0.5\times(150/800)^2\times(2670/500-1)+0.09626\\ &= 0.17255\end{aligned}$$

$$\begin{aligned}\bar{Y} &= (D_s/D)(b_{eff}/b\text{ - }1)+Y\\ &= (150/800)\times(2670/500-1)+0.12834\\ &= 0.94209\end{aligned}$$

$$\begin{aligned}\bar{k} &= (0.5+\bar{X})(1+\bar{Y})\\ &= (0.5+0.17255)/(1+0.94209)\\ &= 0.34630\end{aligned}$$

$$\begin{aligned}x &= \bar{k}D\\ &= 0.34630\times 800\\ &= 277\text{ mm}\end{aligned}$$

A further equation to that given in Fig. 5.4 is for the second moment of area of the uncracked section, i.e. repeating Eq. 7.2(2):

$$I_{uncr} = \kappa bD^3/12$$

where –

$$\kappa = 1+(\frac{b_{eff}}{b}-1)(\frac{D_s}{D})^3+12(0.5-\bar{k})^2+12(\frac{b_{eff}}{b}-1)(\frac{D_s}{D})(\bar{k}-0.5\frac{D_s}{D})^2$$
$$+12(n-1)\bar{p}\left[\left(\frac{d_{bot}}{D}-\bar{k}\right)^2+\frac{A_{s.top}}{A_{s.bot}}\left\{\bar{k}-\frac{(D-d_{top})}{D}\right\}^2\right]$$

It follows that:

$$\kappa = 1+(\frac{2670}{500}-1)(\frac{150}{800})^3+12\times(0.5-0.34630)^2+12\times(\frac{2670}{500}-1)(\frac{150}{800})(0.34630-0.5\times\frac{150}{800})^2$$
$$+12\times(7.9-1)\times 0.0155\times\left[\left(\frac{710}{800}-0.34630\right)^2+\frac{1240}{6200}\left\{0.34630-\frac{(800-750)}{800}\right\}^2\right]$$
$$= 2.332$$

Thus,

$$\begin{aligned} I_{\text{uncr}} &= \kappa b D^3/12 \\ &= 2.332 \times 500 \times 800^3/12 \\ &= 0.0497 \times 10^{12}\ \text{mm}^4 \end{aligned}$$

It follows from Eq. 5.3(1), noting that $Z=I_{uncr}/(D\text{-}x)$, that:

$$\begin{aligned} (M_{uo})_{\min} &= 1.2 Z f'_{cf} \\ &= 1.2 \times 0.0497 \times 10^{12}/(800-277) \times 0.6\sqrt{25}/10^6 \\ &= 342\ \text{kNm} \end{aligned}$$

According to Warner et al., the nominal positive moment capacity M_{uo}=1365/ϕ=1365/0.8=1706 kNm which is greater than $(M_{uo})_{min}$. Therefore, the minimum strength requirement is clearly satisfied.

Further from Fig. 5.4, noting that x>D_s, it can be written that the area of concrete in tension immediately prior to cracking, A_{ct}, equals:

$$\begin{aligned} A_{ct} &= b(D-x) \\ &= 500 \times (800-277) \\ &= 261500\ \text{mm}^2 \end{aligned}$$

The minimum area of tension reinforcement, $A_{st.min}$, can now be calculated using Eq. 5.3(3). In this equation, k_s=0.6 for flexure, A_{ct}=261500 mm^2 and f_s=185 MPa (from Table 8.6.1(A) of AS 3600-2000 for Y28 bars), whereby:

$$\begin{aligned} A_{st.min} &= 3k_s A_{ct}/f_s \\ &= 3 \times 0.6 \times 261500/185 \\ &= 2544\ \text{mm}^2 \end{aligned}$$

Since the actual area of tension steel at the peak positive moment section, A_{st}=6200 mm^2, the minimum reinforcement requirement for crack control is clearly satisfied.

The next step is to calculate the stresses in the tension reinforcement after cracking, f_{scr}, under the action of M^*_s and $M^*_{s.1}$. For this purpose, Fig. 5.8 will be followed:

$$\begin{aligned} p &= A_{st}/(bd) \\ &= 6200/(500 \times 710) \\ &= 0.01746 \\ X &= np[1+(n-1)A_{sc}d_{sc}/(nA_{st}d)] \\ &= 7.9 \times 0.01746 \times [1+(7.9-1) \times 1240 \times 50/(7.9 \times 6200 \times 710)] \\ &= 0.1397 \\ Y &= np[1+(n-1)A_{sc}/(nA_{st})] \\ &= 7.9 \times 0.01746 \times [1+(7.9-1) \times 1240/(7.9 \times 6200)] \\ &= 0.1620 \\ \overline{X} &= (D_s/d)^2(b_{eff}/b - 1) + 2X \\ &= (150/710)^2 \times (2670/500-1) + 2 \times 0.1397 \\ &= 0.4731 \end{aligned}$$

$$\begin{aligned} \overline{Y} &= (D_s/d)(b_{eff}/b - 1) + Y \\ &= (150/710) \times (2670/500-1) + 0.1620 \\ &= 1.0790 \end{aligned}$$

$$k = -\overline{Y} + \sqrt{\overline{Y}^2 + \overline{X}}$$
$$= -1.0790 + \sqrt{1.0790^2 + 0.4731}$$
$$= 0.2006$$
$$d_n = kd$$
$$= 0.2006 \times 710$$
$$= 142 \text{ mm}$$

However, d_n<D_s, which invalidates using Fig. 5.8. As stated in Note 1 of Fig. 5.8, the equations in Fig. 5.7 will have to be used instead, replacing b with b_{eff}. It follows that:

$$p = A_{st} /(b_{eff} d)$$
$$= 6200 /(2670 \times 710)$$
$$= 0.003271$$
$$X = np[1 + (n-1)A_{sc} d_{sc} /(nA_{st} d)]$$
$$= 7.9 \times 0.003271 \times [1 + (7.9-1) \times 1240 \times 50 /(7.9 \times 6200 \times 710)]$$
$$= 0.02616$$
$$Y = np[1 + (n-1)A_{sc} /(nA_{st})]$$
$$= 7.9 \times 0.003271 \times [1 + (7.9-1) \times 1240 /(7.9 \times 6200)]$$
$$= 0.03035$$
$$k = -Y + \sqrt{Y^2 + 2X}$$
$$= -0.03035 + \sqrt{0.03035^2 + 2 \times 0.02616}$$
$$= 0.2004$$
$$d_n = kd$$
$$= 0.2004 \times 710$$
$$= 142 \text{ mm}$$

which agrees with before, and now calculating I_{cr}:

$$\kappa = 4k^3 + 12np(1-k)^2 + 12(n-1)p\frac{A_{sc}}{A_{st}}(k - \frac{d_{sc}}{d})^2$$
$$= 4 \times 0.2006^3 + 12 \times 7.9 \times 0.003271 \times (1-0.2006)^2 + 12 \times (7.9-1) \times 0.003271 \times \frac{1240}{6200} \times (0.2006 - \frac{50}{710})^2$$
$$= 0.23134$$

$$I_{cr} = \kappa b_{eff} d^3 /12$$
$$= 0.23134 \times 2670 \times 710^3 /12$$
$$= 0.01842 \times 10^{12} \text{ mm}^4$$

$$f_{scr.1} = nM^*_{s.1}(d - d_n)/I_{cr}$$
$$= 7.9 \times 835 \times 10^6 \times (710 - 142)/(0.01842 \times 10^{12})$$
$$= 203 \text{ MPa}$$

$$f_{scr} = nM^*_s(d - d_n)/I_{cr}$$
$$= 7.9 \times 730 \times 10^6 \times (710 - 142)/(0.01842 \times 10^{12})$$
$$= 178 \text{ MPa}$$

It follows that because $f_{scr.1}$<0.8f_{sy} (=320 MPa), and f_{scr}<185 MPa from Table 8.6.1(A) of AS 3600-2000, crack control is satisfactory.

Considering the spacing of the bottom tension bars, in accordance with the advice given in Section 5.4, the minimum centre-to-centre distance of adjacent bars should be about 60 mm, i.e. a clear distance of at least 30 mm between bars. This requirement is met by placing the bars in two layers as shown in Fig. 7.1, and explains why a single layer was not chosen. The value of the effective depth *d* (=710 mm) assumed in the calculations is conservative if the two equal layers of reinforcement are separated by say a 32 mm spacer bar.

This completes the check of the peak positive moment section with the original design using 400 MPa bars.

Note: Computer program 500PLUS-BCC has been run with the covers and gap between layers appropriately assigned to give the assumed values of effective depth. The results show that the reinforcement arrangements in Fig. 7.1 still hold, except that only 9Y28 bars are required in the bottom face. The compressive reinforcement was included in the runs. The results confirm that the area of the bottom tension steel is governed by strength, while serviceability controls the area of tension steel in the negative moment region.

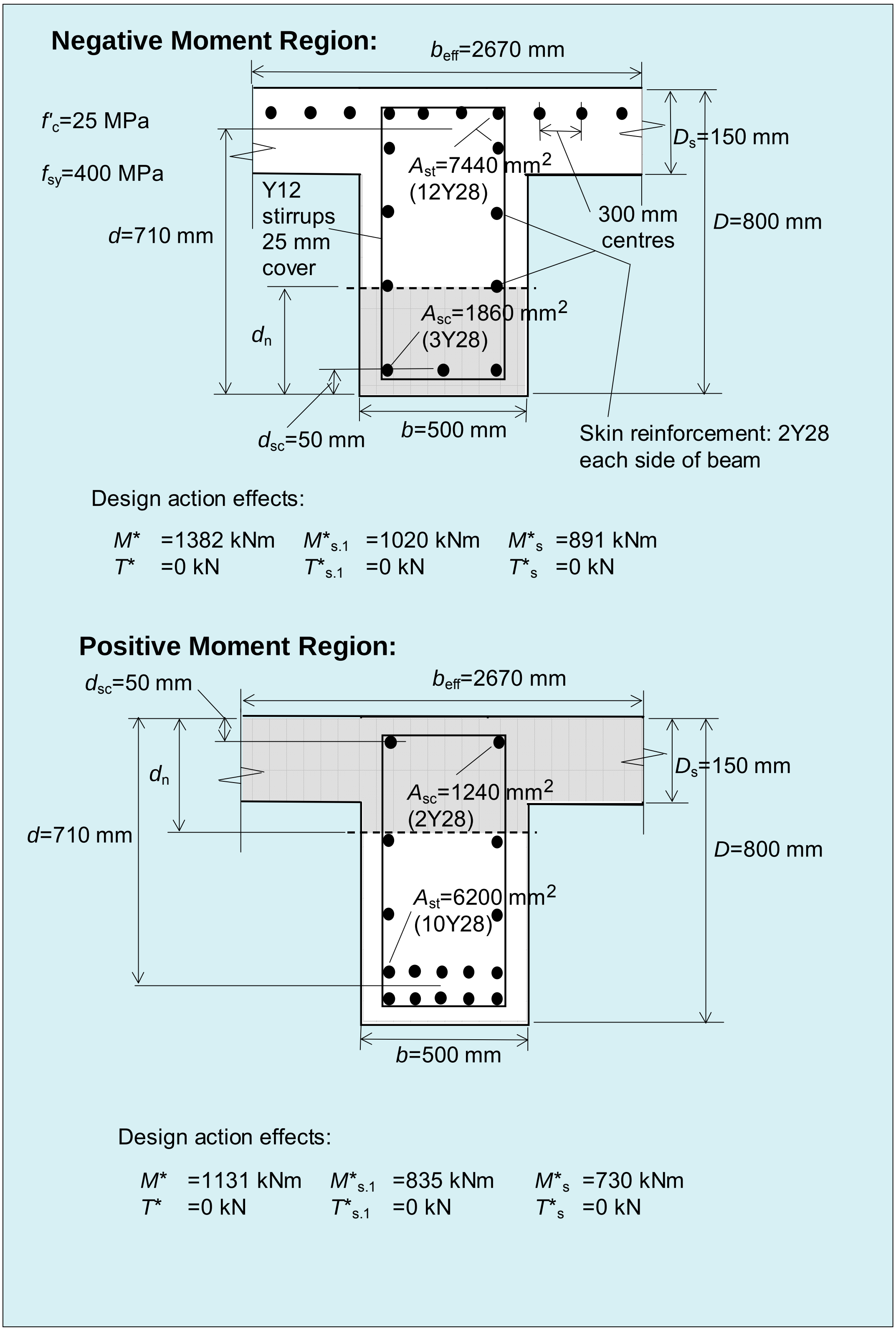


Figure 7.1 Crack Control Design of T-Beam from Ref. [1] – Original Reinforcement Layouts

Note: In accordance with the preceding calculations, it is recommended that in the negative moment region all 12 Y28 bars are placed in the top face (one layer), and distributed across the whole of the effective width. This change in detailing is required for crack control, and meets a requirement that the maximum centre-to-centre spacing does not exceed 250 mm.

Part 2 – Improved design using 500PLUS® Rebar (see Fig. 7.2)

The amount of tension reinforcement in the negative and positive moment regions will be reviewed during its conversion to 500PLUS Rebar. The objective is to minimise the cross-sectional area of steel while still maintaining crack control.

For completeness, a full set of calculations as in Part 1 will be presented. This includes checking the minimum strength requirement according to the procedure shown in Fig. 4.2. The flowcharts in Figs 4.3, 4.4 and 4.5 are followed as appropriate. With tension assumed zero, it is clear that there is a state of flexure throughout the beam.

Finally, some comments will be made about the effect that the significantly reduced steel area has on flexural stiffness, although strictly this subject is beyond the scope of this booklet.

As a consequence of using 500PLUS Rebar instead of 400 MPa reinforcement, it will be assumed at the outset that it is possible for all the tension steel at the peak positive and negative moment sections to be placed in one rather than two layers. This will clearly not be a problem for the steel in the top face. Moreover, the preceding calculations for the 400 MPa top tension steel have shown that the stress in these bars can be expected to increase very approximately to about 194×(500/400)=243 MPa under the action of M_s^*. It is clear from Tables 8.6.1(A) and (B) of AS 3600-2000 that this will require the original Y28 bars to reduce in size to N24 500PLUS Rebar. (The 500PLUS Rebar will be designated N24 in accordance with AS 3600-2000 to indicate Class N or normal ductility, 24 mm diameter reinforcing bars.) This will reduce the bar spacing to about 200 mm as shown in Fig. 7.2, and increase the allowable maximum steel stress to 240 MPa (which the design calculations below will confirm is satisfactory).

For the tension reinforcement in the positive moment region to be placed in one layer as shown in Fig. 7.2, spacing limitations require that the number of 28 mm diameter bars will need to reduce significantly. It will be noticed that the compression reinforcement in this region has been changed from two Y28 bars to 2N24 500PLUS Rebar, thus allowing these bars to be continuous along the beam.

By moving to single layers of reinforcement, the effective depth, d, of the tension steel in both regions will be assumed to increase from 710 mm to 745 mm, rounding down slightly to ensure that d will not be overestimated while still achieving 25 mm of concrete cover. To be consistent with this assumption, the calculations will use d_{sc}=55 mm. Consequently, the improved reinforcement layouts shown in Fig. 7.2 will be trialed for crack control. Detailed calculations show that they have sufficient bending strength, with $\phi M_{uo} > M^*$ for negative and positive bending, respectively. Specifically, ϕM_{uo}=1540 kNm (>M^*=1382 kNm) in the negative moment region, and ϕM_{uo}=1261 kNm (>M^*=1131 kNm) in the positive moment region. (These values of ϕM_{uo} are also given in Figs 6.4 and 6.6, and can thus be obtained from program 500PLUS-BCC.) Once again, the skin reinforcement will be ignored in the calculations, noting that it has also been reduced in area by specifying N16 500PLUS Rebar (instead of the Y28 bars), which is all that is required by Clause 8.6.3 of AS 3600-2000.

Negative moment region

The uncracked section properties are calculated as follows. Firstly, from Fig. 5.4:

$$\begin{aligned}\bar{p} &= A_{s.bot}/(bD) \\ &= 1860/(500 \times 800) \\ &= 0.00465\end{aligned}$$

$$d_{bot} = d_{top} = 745 \text{ mm}$$

$$X = (n-1)\bar{p}(d_{bot}/D)[1 + A_{s.top}(D - d_{top})/(A_{s.bot}d_{bot})]$$
$$= (7.9-1)\times 0.00465\times(745/800)\times[1+5850\times(800-745)/(1860\times 745)]$$
$$= 0.036817$$
$$Y = (n-1)\bar{p}(1 + A_{s.top}/A_{s.bot})$$
$$= (7.9-1)\times 0.00465\times(1+5850/1860)$$
$$= 0.13300$$
$$\bar{X} = 0.5(D_s/D)^2(b_{eff}/b\text{ -}1) + X$$
$$= 0.5\times(150/800)^2\times(2670/500-1)+0.036817$$
$$= 0.113106$$
$$\bar{Y} = (D_s/D)(b_{eff}/b\text{ -}1) + Y$$
$$= (150/800)\times(2670/500-1)+0.13300$$
$$= 0.94675$$
$$\bar{k} = (0.5+\bar{X})(1+\bar{Y})$$
$$= (0.5+0.113106)/(1+0.94675)$$
$$= 0.31494$$
$$x = \bar{k}D$$
$$= 0.31494\times 800$$
$$= 252\text{ mm}$$

A further equation to that given in Fig. 5.4 is for the second moment of area of the uncracked section, i.e. repeating Eq. 7.2(2):

$$I_{uncr} = \kappa b D^3/12$$

where –

$$\kappa = 1 + (\frac{b_{eff}}{b}-1)(\frac{D_s}{D})^3 + 12(0.5-\bar{k})^2 + 12(\frac{b_{eff}}{b}-1)(\frac{D_s}{D})(\bar{k}-0.5\frac{D_s}{D})^2$$
$$+12(n-1)\bar{p}\left[\left(\frac{d_{bot}}{D}-\bar{k}\right)^2 + \frac{A_{s.top}}{A_{s.bot}}\left\{\bar{k}-\frac{(D-d_{top})}{D}\right\}^2\right]$$

It follows that:

$$\kappa = 1 + (\frac{2670}{500}-1)(\frac{150}{800})^3 + 12\times(0.5-0.31494)^2 + 12\times(\frac{2670}{500}-1)(\frac{150}{800})(0.31494-0.5\times\frac{150}{800})^2$$
$$+12\times(7.9-1)\times 0.00465\times\left[\left(\frac{745}{800}-0.31494\right)^2 + \frac{5850}{1860}\left\{0.31494-\frac{(800-745)}{800}\right\}^2\right]$$
$$= 2.137$$

Thus,

$$I_{uncr} = \kappa b D^3/12$$
$$= 2.137\times 500\times 800^3/12$$
$$= 0.04560\times 10^{12}\text{ mm}^4$$

It follows from Eq. 5.3(1), noting that $Z = I_{uncr}/x$, that:

$$
\begin{aligned}
(M_{uo})_{\min} &= 1.2Zf'_{cf} \\
&= 1.2\times(0.04560\times10^{12}/252)\times0.6\sqrt{25}/10^{6} \\
&= 652\text{ kNm}
\end{aligned}
$$

As previously stated, the nominal negative moment capacity M_{uo} = 1540/ϕ=1540/0.8=1925 kNm which is greater than $(M_{uo})_{min}$, and the minimum strength requirement is clearly satisfied.

Further from Fig. 5.4, noting that x>D_s, it can be written that the area of concrete in tension immediately prior to cracking, A_{ct}, equals:

$$
\begin{aligned}
A_{ct} &= b_{eff}D_s + b(x - D_s) \\
&= 2670\times150 + 500\times(252-150) \\
&= 451500\text{ mm}^2
\end{aligned}
$$

The minimum area of tension reinforcement, $A_{st.min}$, can now be calculated using Eq. 5.3(3). In this equation, k_s=0.6 for flexure, A_{ct}=451500 mm^2 and f_s=210 MPa (from Table 8.6.1(A) of AS 3600-2000 for N24 500PLUS Rebar), whereby:

$$
\begin{aligned}
A_{st.min} &= 3k_s A_{ct}/f_s \\
&= 3\times0.6\times451500/210 \\
&= 3870\text{ mm}^2
\end{aligned}
$$

Since the actual area of tension steel at the peak negative moment section, A_{st}=5580 mm^2, the minimum reinforcement requirement for crack control is clearly satisfied.

The next step is to calculate the stresses in the tension reinforcement after cracking, f_{scr}, under the action of M^*_s and $M^*_{s.1}$. For this purpose, Fig. 5.9 will be followed:

$$
\begin{aligned}
p &= A_{st}/(bd) \\
&= 5850/(500\times745) \\
&= 0.01571 \\
X &= np[1+(n-1)A_{sc}d_{sc}/(nA_{st}d)] \\
&= 7.9\times0.01571\times[1+(7.9-1)\times1860\times55/(7.9\times5850\times745)] \\
&= 0.1267 \\
Y &= np[1+(n-1)A_{sc}/(nA_{st})] \\
&= 7.9\times0.01571\times[1+(7.9-1)\times1860/(7.9\times5850)] \\
&= 0.1586 \\
k &= -Y+\sqrt{Y^2+2X} \\
&= -0.1586+\sqrt{0.1586^2+2\times0.1267} \\
&= 0.3692 \\
d_n &= kd \\
&= 0.3692\times745 \\
&= 275\text{ mm}
\end{aligned}
$$

Note: because d_n<(D - D_s) (=650 mm), use of the equations in Fig. 5.9 is valid.

$$
\begin{aligned}
\kappa &= 4k^3 + 12np(1-k)^2 + 12(n-1)p\frac{A_{sc}}{A_{st}}(k-\frac{d_{sc}}{d})^2 \\
&= 4\times0.3692^3 + 12\times7.9\times0.01571\times(1-0.3692)^2 + 12\times(7.9-1)\times0.01571\times\frac{1860}{5850}\times(0.3692-\frac{55}{745})^2 \\
&= 0.8300
\end{aligned}
$$

$$\begin{aligned} I_{cr} &= \kappa b d^3 / 12 \\ &= 0.8300 \times 500 \times 745^3 / 12 \\ &= 0.01431 \times 10^{12} \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} f_{scr.1} &= n M^*_{s.1} (d - d_n) / I_{cr} \\ &= 7.9 \times 1020 \times 10^6 \times (745 - 275) / (0.01431 \times 10^{12}) \\ &= 265 \text{ MPa} \end{aligned}$$

$$\begin{aligned} f_{scr} &= n M^*_{s} (d - d_n) / I_{cr} \\ &= 7.9 \times 891 \times 10^6 \times (745 - 275) / (0.01431 \times 10^{12}) \\ &= 231 \text{ MPa} \end{aligned}$$

It follows that because $f_{scr.1}$<0.8f_{sy} (=400 MPa), yielding the reinforcement under service loads is not possible, and the design is satisfactory in this respect.

However, f_{scr}>210 MPa, which is the maximum allowable stress from Table 8.6.1(A) of AS 3600-2000 for N24 500PLUS Rebar. However, in accordance with Clause 8.6.1(e) of AS 3600-2000, crack control can be achieved by reducing the spacing of the bars. This must be done across the whole of the effective width of the beam. In accordance with the equation in Table 8.6.1(B), the maximum acceptable centre-to-centre spacing equals (400-231)/0.8 = 211 mm. As explained at the start of these calculations, this is why N24 500PLUS Rebar was chosen, which confirms that the design is satisfactory.

This completes the check of the peak negative moment section with the improved design using 500PLUS Rebar. The cross-sectional area of the main tension steel has been reduced from 7440 to 5850 mm^2, representing a reduction of 21 per cent on the original design, slightly more than the full 20 per cent benefit available from the increase in steel grade. This is partly due to the increased effective depth of the reinforcement, which was achieved by placing the top bars in one layer, and has improved the overall efficiency of the section. Also, the maximum bar spacing of 210 mm requires 13N24 500PLUS Rebar, while 12N24 500PLUS Rebar would have been satisfactory for bending strength, so crack control has still had an effect on the final design.

The computer program 500PLUS-BCC has been run (see Fig. 6.5), and the results show that in fact 17N20 500PLUS Rebar (A_{st}=5270 mm^2) are satisfactory. Thus, there is a significant benefit in further reducing the bar size. This represents a reduction of 29 per cent on the original design. (See the note just before Fig. 7.1, where it is explained 500PLUS-BCC gives exactly the same result for 400 MPa steel as shown in Fig. 7.1 for the negative moment region.) It is the maximum reduction available as a consequence of the increased steel strength and greater effective depth, and there is no need to reduce the bar size further.

Positive moment region

The uncracked section properties are calculated as follows. Firstly, from Fig. 5.4:

$$\begin{aligned} \overline{p} &= A_{s.bot} / (bD) \\ &= 4340 / (500 \times 800) \\ &= 0.01085 \end{aligned}$$

$d_{bot} = d_{top}$ = 745 mm

$$\begin{aligned} X &= (n-1)\overline{p}(d_{bot}/D)[1 + A_{s.top}(D - d_{top})/(A_{s.bot} d_{bot})] \\ &= (7.9-1) \times 0.01085 \times (745/800) \times [1 + 900 \times (800 - 745)/(4340 \times 745)] \\ &= 0.07079 \end{aligned}$$

$$\bar{Y} = (n-1)\bar{p}(1 + A_{s.top} / A_{s.bot})$$
$$= (7.9-1) \times 0.01085 \times (1 + 900/4340)$$
$$= 0.09039$$
$$\bar{X} = 0.5(D_s / D)^2 (b_{eff} / b - 1) + X$$
$$= 0.5 \times (150/800)^2 \times (2670/500 - 1) + 0.07079$$
$$= 0.14708$$
$$\bar{Y} = (D_s / D)(b_{eff} / b - 1) + Y$$
$$= (150/800) \times (2670/500 - 1) + 0.09039$$
$$= 0.90414$$
$$\bar{k} = (0.5 + \bar{X})(1 + \bar{Y})$$
$$= (0.5 + 0.14780)/(1 + 0.90414)$$
$$= 0.3400$$
$$x = \bar{k}D$$
$$= 0.3400 \times 800$$
$$= 272 \text{ mm}$$

A further equation to that given in Fig. 5.4 is for the second moment of area of the uncracked section, i.e. repeating Eq. 7.2(2):

$$I_{uncr} = \kappa b D^3 / 12$$

where –

$$\kappa = 1 + (\frac{b_{eff}}{b} - 1)(\frac{D_s}{D})^3 + 12(0.5 - \bar{k})^2 + 12(\frac{b_{eff}}{b} - 1)(\frac{D_s}{D})(\bar{k} - 0.5\frac{D_s}{D})^2$$
$$+12(n-1)\bar{p}\left[\left(\frac{d_{bot}}{D} - \bar{k}\right)^2 + \frac{A_{s.top}}{A_{s.bot}}\left\{\bar{k} - \frac{(D - d_{top})}{D}\right\}^2\right]$$

It follows that:

$$\kappa = 1 + (\frac{2670}{500} - 1)(\frac{150}{800})^3 + 12 \times (0.5 - 0.34021)^2 + 12 \times (\frac{2670}{500} - 1)(\frac{150}{800})(0.34021 - 0.5 \times \frac{150}{800})^2$$
$$+12 \times (7.9 - 1) \times 0.01085 \times \left[\left(\frac{745}{800} - 0.34021\right)^2 + \frac{900}{4340}\left\{0.34021 - \frac{(800 - 745)}{800}\right\}^2\right]$$
$$= 2.256$$

Thus,

$$I_{uncr} = \kappa b D^3 / 12$$
$$= 2.256 \times 500 \times 800^3 / 12$$
$$= 0.04813 \times 10^{12} \text{ mm}^4$$

It follows from Eq. 5.3(1), noting that $Z = I_{uncr}/(D-x)$, that:

$$(M_{uo})_{min} = 1.2 Z f'_{cf}$$
$$= 1.2 \times 0.04813 \times 10^{12} / (800 - 272) \times 0.6\sqrt{25} / 10^6$$
$$= 328 \text{ kNm}$$

As previously stated, the nominal positive moment capacity M_{uo}=1261/ϕ=1261/0.8=1576 kNm which is greater than $(M_{uo})_{min}$, and the minimum strength requirement is clearly satisfied.

Further from Fig. 5.4, noting that x>D_s, it can be written that the area of concrete in tension immediately prior to cracking, A_{ct}, equals:

$$\begin{aligned} A_{ct} &= b(D-x) \\ &= 500\times(800-272) \\ &= 264000\ \text{mm}^2 \end{aligned}$$

The minimum area of tension reinforcement, $A_{st.min}$, can now be calculated using Eq. 5.3(3). In this equation, k_s=0.6 for flexure, A_{ct}=264500 mm^2 and f_s=185 MPa (from Table 8.6.1(A) of AS 3600-2000 for Y28 bars), whereby:

$$\begin{aligned} A_{st.min} &= 3k_s A_{ct}/f_s \\ &= 3\times0.6\times264000/185 \\ &= 2569\ \text{mm}^2 \end{aligned}$$

Since the actual area of tension steel at the peak positive moment section, A_{st}=4340 mm^2, the minimum reinforcement requirement for crack control is clearly satisfied.

The next step is to calculate the stresses in the tension reinforcement after cracking, f_{scr}, under the action of M_s^* and $M_{s.1}^*$. For this purpose, Fig. 5.7 will be followed, noting that it can be expected as for the original design that d_n<D_s in Fig. 5.8, which would have been used otherwise. Replacing b with b_{eff} in Fig. 5.7 gives:

$$\begin{aligned} p &= A_{st}/(b_{eff}d) \\ &= 4340/(2670\times745) \\ &= 0.002182 \\ X &= np[1+(n-1)A_{sc}d_{sc}/(nA_{st}d)] \\ &= 7.9\times0.002182\times[1+(7.9-1)\times900\times55/(7.9\times4340\times745)] \\ &= 0.01747 \\ Y &= np[1+(n-1)A_{sc}/(nA_{st})] \\ &= 7.9\times0.002182\times[1+(7.9-1)\times900/(7.9\times4340)] \\ &= 0.02036 \\ k &= -Y+\sqrt{Y^2+2X} \\ &= -0.02036+\sqrt{0.02036^2+2\times0.01747} \\ &= 0.1677 \\ d_n &= kd \\ &= 0.1677\times745 \\ &= 125\ \text{mm} \end{aligned}$$

$$\begin{aligned} \kappa &= 4k^3+12np(1-k)^2+12(n-1)p\frac{A_{sc}}{A_{st}}(k-\frac{d_{sc}}{d})^2 \\ &= 4\times0.1677^3+12\times7.9\times0.002182\times(1-0.1677)^2+12\times(7.9-1)\times0.002182\times\frac{900}{4340}\times(0.1677-\frac{55}{745})^2 \\ &= 0.1625 \end{aligned}$$

$$I_{cr} = \kappa b_{eff} d^3 / 12$$
$$= 0.1625 \times 2670 \times 745^3 / 12$$
$$= 0.01497 \times 10^{12} \text{ mm}^4$$

$$f_{scr.1} = nM^*_{s.1}(d - d_n)/I_{cr}$$
$$= 7.9 \times 835 \times 10^6 \times (745 - 125)/(0.01497 \times 10^{12})$$
$$= 274 \text{ MPa}$$

$$f_{scr} = nM^*_{s}(d - d_n)/I_{cr}$$
$$= 7.9 \times 730 \times 10^6 \times (745 - 125)/(0.01497 \times 10^{12})$$
$$= 239 \text{ MPa}$$

It follows that because $f_{scr.1}$<0.8f_{sy} (=400 MPa), yielding the reinforcement under service loads is not possible, and the design is satisfactory in this respect.

However, f_{scr}>185 MPa, which is the maximum allowable stress from Table 8.6.1(A) of AS 3600-2000 for N28 500PLUS Rebar. In accordance with Clause 8.6.1(e) of AS 3600-2000, crack control can be achieved by reducing the spacing of the bars. In accordance with the equation in Table 8.6.1(B), the maximum acceptable centre-to-centre spacing equals (400-239)/0.8 = 201 mm. Of course this is easily satisfied in the bottom of the beam, and therefore the design is satisfactory with regard to crack control.

This completes the check of the peak positive moment section with the improved design using 500PLUS Rebar. The cross-sectional area of the main tension steel has been reduced from 6200 to 4340 mm^2, representing a reduction of 30 per cent on the original design, more than the full 20 per cent benefit available from the increase in steel grade. However, as pointed at in the note just prior to Fig. 7.1, program 500PLUS-BCC shows that only 9Y28 (=5580 mm^2) were actually required in the original design. Thus the reduction should have only been 22 per cent. It has been shown above that the extra 2 per cent essentially comes about by being able to reduce the number of bars, which allows all the main tension reinforcement to be placed in one layer. This increases the effective depth of this reinforcement, and further improves the overall efficiency of the section. Although the steel tensile stresses in the positive moment region increased by 34% compared with the original design in Fig. 7.1, the beam shown in Fig. 7.2 is entirely satisfactory with regard to crack control.

Flexural stiffness

The effect that the changes to the original design have had on the flexural stiffness of the two-span continuous beam is briefly discussed.

The values of second moment of area for the uncracked and cracked states that were computed during the course of this example have been used to calculate the average short-term flexural stiffness of the two-span beam, both for the original design and the new design. These calculations show that as a consequence of the reduction in steel area, the average short-term flexural stiffness has been reduced by 9 per cent compared with the original design.

However, the beam was originally proportioned using the deemed-to-comply, span-to-depth ratio equation in Section 8.5.4 of AS 3600-1994. The average second moment of area equalled [1]:

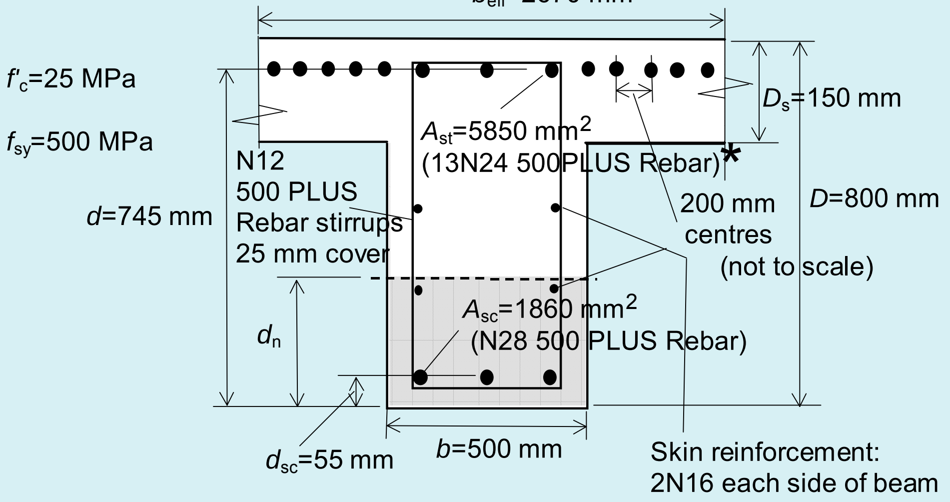
$$I_{av} = k_1 b_{ef} d^3$$
$$= 0.045 \times (0.7 + 0.3\frac{b}{b_{ef}})^3 b_{ef} d^3$$
$$= 0.045 \times (0.7 + 0.3 \times \frac{500}{2670})^3 \times 2670 \times 710^3$$
$$= 0.01863 \times 10^{12} \text{ mm}^4$$

However, the detailed calculations carried out using the values of second moment of area I_{uncr} and I_{cr} in the example show that for the improved design:

$$I_{av} = 0.01903 \times 10^{12} \text{ mm}^4$$

Therefore, despite the significant reduction in steel area that resulted from moving to 500PLUS Rebar, deflection control of the beam is still satisfactory according to AS 3600.

Negative Moment Region:

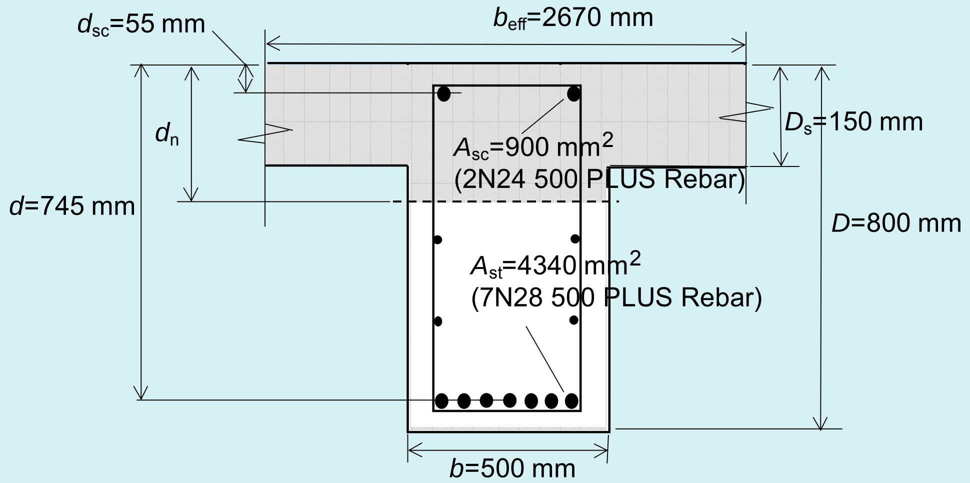


Design action effects:

M^*	=1382 kNm	$M^*_{s.1}$	=1020 kNm	M^*_s	=891 kNm
T^*	=0 kN	$T^*_{s.1}$	=0 kN	T^*_s	=0 kN

* Running computer program 500PLUS-BCC shows that by further reducing the top bar diameter to 20 mm (see Fig. 6.5): A_{st}=5270 mm^2 (17N20 500PLUS Rebar) is in fact sufficient.

Positive Moment Region:



Design action effects:

M^*	=1131 kNm	$M^*_{s.1}$	=835 kNm	M^*_s	=730 kNm
T^*	=0 kN	$T^*_{s.1}$	=0 kN	T^*_s	=0 kN

Figure 7.2 Crack Control Design of T-Beam from Ref. [1] – Improved Reinforcement Layouts

7.3 Example 2 - Multi-Span Continuous T-Beam Designed to AS 3600-2000 Using 500PLUS® Rebar

This example will be used to show the influence that the following factors can have on the design of a continuous reinforced-concrete beam when crack control is important:

(i) bar diameter;

(ii) moment redistribution at the strength limit state;

(iii) calculation of design bending moments under service loads; and

(iv) load intensity and type (i.e. office vs compactus).

As well as showing good design practice, the example aims to highlight to designers the types of circumstances when the full benefit of the high strength 500PLUS Rebar can be achieved. This requires that strength considerations will govern the cross-sectional area of the main reinforcement at critical sections. Then, as shown in the previous example, a reduction of 20 per cent in steel area can be achieved compared with using 400 MPa bars. The significant advantage of using computer program 500PLUS-BCC™ (see Section 6) during the design process, particularly when choosing the final bar sizes, will also be illustrated. The differences between the designs using 500PLUS Rebar compared with those had 400 MPa steel been used will be highlighted using the software, which can be used very simply for such purposes.

Appendix C has been prepared to assist with this design example. It contains rules for calculating design bending moments under service loads. For simplicity, all loads on the continuous beam will be assumed uniformly distributed. No account will be taken here of any effect of two-way slab action on load transfer to the beams.

In addition, design for vertical shear, torsion, etc. will not be considered.

The example will be broken into the following parts:

Part 1 – Design criteria

Part 2 – Preliminary sizing

Part 3 – Calculation of design bending moments

Part 4 – Design for bending strength

Part 5 – Effect of bar diameter and steel grade on crack control design

Part 1 – Design criteria

A typical internal span of an interior region of a multi-span continuous beam will be designed for crack control in accordance with the design rules proposed for AS 3600-2000. This implies that crack widths will generally be kept to less than 0.3 mm under serviceability conditions. Although the beam is internal (exposure classification A1 in AS 3600-1994), it is still considered desirable to limit crack widths to avoid problems with floor finishes and the chance of client concerns. The beam will only be designed for the effects of direct loading. Therefore, a state of flexure will be assumed to apply at critical sections when designing for crack control.

A bandbeam in a conventional bandbeam and slab arrangement [20] will be designed. The column grid is 8.1 metres (in the direction of the slab span) × 9.4 metres (in the direction of the bandbeam span). End spans are assumed to be 0.8 times the internal span. The bandbeam is 2400 mm wide with a rectangular section, leaving a clear span of 5700 mm for the slab. The columns are 600×600 mm, being located in the lower floors of a multi-storey building.

Construction is monolithic with the bandbeams cast together with the slabs, all fully supported on conventional formwork and falsework, i.e. "fully-propped" construction. (Note: even if the slabs were cast using precast elements or a permanent steel formwork system[4] such that they were unpropped

[4] OneSteel Reinforcing now produce TRUSSDEK™, which is a permanent steel formwork system that has particularly long-spanning capabilities, and can significantly reduce the amount of falsework, reinforcement, on-site labour, etc. and speed up construction in this type of application.

or partially propped during construction, this would not necessarily affect the crack control design of the bandbeam.)

For simplicity, construction loads that occur after the falsework has been removed will not be considered critical, and the bandbeam will only be designed for the in-service condition. The design loads for strength design and serviceability design can be calculated using the following information:

Superimposed dead load, G_{sup} = 1.3 kPa

Live load, Q = 4.0 kPa – office areas

Live load, Q = 10.0 kPa – storage (compactus) areas

(Note: in accordance with AS 1170.1, ψ_s=0.7 and 1.0 and ψ_l=0.4 and 0.6 for office and storage areas, respectively. Also, at the request of the client, live load reduction will not be considered for the office areas, while it is not appropriate for the compactus areas.)

Concrete density, ρ_c = 2400 kg/m^3

Allowance for reinforcing steel = 100 kg/m^3

(Note: this estimate will not be readjusted in light of the final design, since ρ_c is only a nominal value. Also, for simplicity displacement of concrete by the steel is ignored.)

Additional design parameters are as follows:

Concrete strength grade, f'_c = 25 MPa (but 32 MPa for compactus load case)

Main steel grade = 500 MPa (500PLUS Rebar)

Main steel ductility class = N (Normal ductility)

(Note: with Class N reinforcement, moment redistribution at the strength limit state is therefore allowed in accordance with Clause 7.6.8.2 of AS 3600-1994, the amount depending on the value of the neutral axis parameter, k_u.)

Deflection limit = L/250 long-term, total deflection

Fire rating (FRL) = 2 hours (120/120/120)

Part 2 – Preliminary sizing

To aid on-site construction efficiency, the depths of the slabs and bandbeams will be kept the same in both the office and compactus areas. Preliminary calculations show that a bandbeam with an overall depth, D, of 350 mm and a slab with a depth, D_s, of 180 mm should be satisfactory with regard to meeting strength and deflection criteria.

The minimum concrete cover to reinforcement will be 20 mm (Table 4.10.3.2 of AS 3600-1994), which also satisfies Fig. 5.4.2(B) of AS 3600-1994 concerning structural adequacy for fire on the bottom face. Accordingly, all main tension and compression reinforcement will be assumed to be centred 50 mm from the nearer adjacent top or bottom face, noting that following on from the previous example, a design objective will be to place reinforcement in one rather than two layers.

Part 3 – Calculation of design bending moments

As mentioned above, Appendix C contains rules for calculating design bending moments under service loads, M_s^*. Relevant parts of Appendix C will also be followed to calculate the design bending moments under ultimate loads, M^*.

Linear elastic analysis will be used for both limit states assuming a prismatic member passing over multiple spans. Rotational restraint from the columns at the supports will conservatively be ignored.

In order to determine the necessary loading cases, in accordance with Paragraph C3 in Appendix C, it is first necessary to determine whether Q>0.75G. If it is, then the pattern loading cases shown in Fig. C3.1 will need to be considered for both the strength and serviceability limit states. It follows that one needs to calculate:

$$G_{cs} = (8.1 \times 0.18 + 2.4 \times 0.17) \times (2.4 + 0.1) \times 9.81$$
$$= 45.76 \text{ kN/m}$$

$$G_{sup} = 8.1 \times 1.3$$
$$= 10.53 \text{ kN/m}$$

$$G = G_{cs} + G_{sup}$$
$$= 56.29 \text{ kN/m}$$

$$0.75G = 42.22 \text{ kN/m}$$

$Q_{off} = 32.4$ kN/m $< 0.75G \Rightarrow$ pattern loading can be ignored in office areas.

$Q_{com} = 81.0$ kN/m $> 0.75G \Rightarrow$ pattern loading cannot be ignored in compactus areas.

Strength limit state

In order to design a typical internal span, a 4-span beam will be modelled with two equal internal spans of 9.4 metres and two equal end spans of 7.52 metres each. The effective width, b_{eff}, is calculated as 3716 mm for an internal span, which takes the same value for serviceability calculations.

In accordance with Paragraph C5, the critical sections for negative design bending moments will be taken at a distance of 210 mm (=0.35×600) from the column centreline.

(a) Case Q=4.0 kPa – No moment redistribution

Analysis shows that at the strength limit state:

- at the critical section adjacent to the peak negative moment section: M^*=767 kNm; and
- at the peak positive moment section: M^*=443 kNm.

(b) Case Q=4.0 kPa – Moment redistribution (see Part 4 below for calculation of amount of redistribution permitted)

Analysis shows that at the strength limit state, based on η=0.3:

- at the critical section adjacent to the peak negative moment section: M^*=494 kNm; and
- at the peak positive moment section: M^*=708 kNm.

(c) Case Q=10.0 kPa – No moment redistribution

Analysis shows that at the strength limit state:

- at the critical section adjacent to the peak negative moment section: M^*=1444 kNm; and
- at the peak positive moment section: M^*=1068 kNm.

Serviceability limit state

In accordance with Paragraph C3(a), the design serviceability bending moments can be calculated directly from the corresponding values for the strength limit state using Eq. 7.2(1), but including the effect of moment redistribution when appropriate, similar to Eq. 4.1(2), viz.:

$$M_s^* = M^* \frac{G + \psi_s Q}{1.25G + 1.5Q} \frac{1}{(1-\eta)} \qquad 7.3(1)$$

In this equation, the degree of moment redistribution, η, is positive if the bending moments at the strength limit state are redistributed downwards, whereby a positive value will be required when calculating peak negative serviceability bending moments. However, a negative value for η will be required in this case when the peak positive serviceability moments are calculated. Values of $M^*_{s.1}$ can be calculated using ψ_s=1.0.

It follows from Eq. 7.3(1) that for Q=4.0 kPa and η=0 (no moment redistribution case):

$$M^*_s = 0.664M^* \quad \text{and} \quad M^*_{s.1} = 0.746M^*$$

while for η=0.3, since the service moments are determined for no redistribution, these values are unchanged.

It follows from Eq. 7.3(1) that for Q=10.0 kPa and η=0 (noting that ψ_s=1.0 when calculating M^*_s):

$$M^*_s = 0.716M^* \quad \text{and} \quad M^*_{s.1} = 0.716M^*$$

(a) Case Q=4.0 kPa – No moment redistribution

It follows from above that:

- at the critical section adjacent to the peak negative moment section:
  M^*=767 kNm; $M^*_{s.1}$=572 kNm; and M^*_s=509 kNm; and
- at the peak positive moment section:
  M^*=443 kNm; $M^*_{s.1}$=331 kNm; and M^*_s=294 kNm.

(b) Case Q=4.0 kPa – Moment redistribution

It follows from above that:

- at the critical section adjacent to the peak negative moment section:
  M^*=494 kNm; $M^*_{s.1}$=572 kNm; and M^*_s=509 kNm; and
- at the peak positive moment section:
  M^*=708 kNm; $M^*_{s.1}$=331 kNm; and M^*_s=294 kNm.

(c) Case Q=10.0 kPa – No moment redistribution

It follows from above that:

- at the critical section adjacent to the peak negative moment section:
  M^*=1444 kNm; $M^*_{s.1}$=1034 kNm; and M^*_s=1034 kNm; and
- at the peak positive moment section:
  M^*=1068 kNm; $M^*_{s.1}$=765 kNm; and M^*_s=765 kNm.

Part 4 – Design for bending strength

As well as satisfying the normal strength requirement that $M^* \le \phi M_{uo}$, design for bending strength entails satisfying the minimum strength requirement at critical sections (see Clause 8.1.4.1 of AS 3600-2000 and Fig. 4.2). One must also ensure that the minimum area of reinforcement for crack control is at least provided (see Clause 8.6.1(a) of AS 3600-2000 and Fig. 4.3).

Computer program 500PLUS-BCC performs all of these design calculations. In order to use the program, the effective section of the bandbeam is needed, i.e. in accordance with Clause 8.8.2 of AS 3600-2000, as stated above b_{ef}=3716 mm. It has also been assumed that the cover to the top and bottom main bars in the bandbeam is 36 mm and 32 mm, respectively, which allows for the presence of top slab reinforcement and/or fitments as appropriate. The side cover to the bottom bars has been assumed to equal 50 mm, which allows for 20 mm cover to N12 500PLUS Rebar stirrups with a 24 mm corner radius and the case of the smallest diameter main bar available. The presence of compression reinforcement in both the critical negative and positive moment regions has generally been conservatively ignored during the analyses. However, this has not been possible for the negative moment region when the beam supports the compactus load. In this case, as noted above, the concrete compressive strength also had to be increased in order to limit the amount of compression reinforcement needed for bending strength.

Part 5 – Effect of bar diameter and steel grade on crack control design

The cross-section details input into program 500PLUS-BCC are shown in Fig. 7.3 (including options of minimum gap between bars of 30 mm, and 30 mm between layers). The results of three computer runs are shown in Figs 7.4 and 7.5 (Q=4.0 kPa, no moment redistribution), Figs 7.6 and 7.7 (Q=4.0 kPa, moment redistribution), and Figs 7.8 and 7.9 (Q=10.0 kPa, no moment redistribution). In each of these runs, f_{sy}=500 MPa appropriate for 500PLUS Rebar. In three accompanying runs (not presented here for brevity), all the same data was used except f_{sy}=400 MPa. The results of all six runs are summarized in Figs 7.10, 7.11 and 7.12, and each graph contains a curve for 400 MPa bar and 500PLUS Rebar.

The graphs generally exhibit the same basic features. For the larger bar diameters, the curves for 400 MPa bar and 500PLUS Rebar are the same, while for the smaller bar diameters, the curves diverge from each other and become approximately horizontal. The area of tensile steel, A_{st}, can be very much greater for the largest bars than for the smallest bars. A horizontal portion of curve implies that strength is governing the minimum value of A_{st}, and when both curves are horizontal, A_{st} for 500PLUS Rebar is about 80 per cent of that for the 400 MPa bars. These are the bar diameters that must be used to get the full benefit of the higher strength grade, while still achieving satisfactory crack control. Readers are left to draw other conclusions from the graphs.

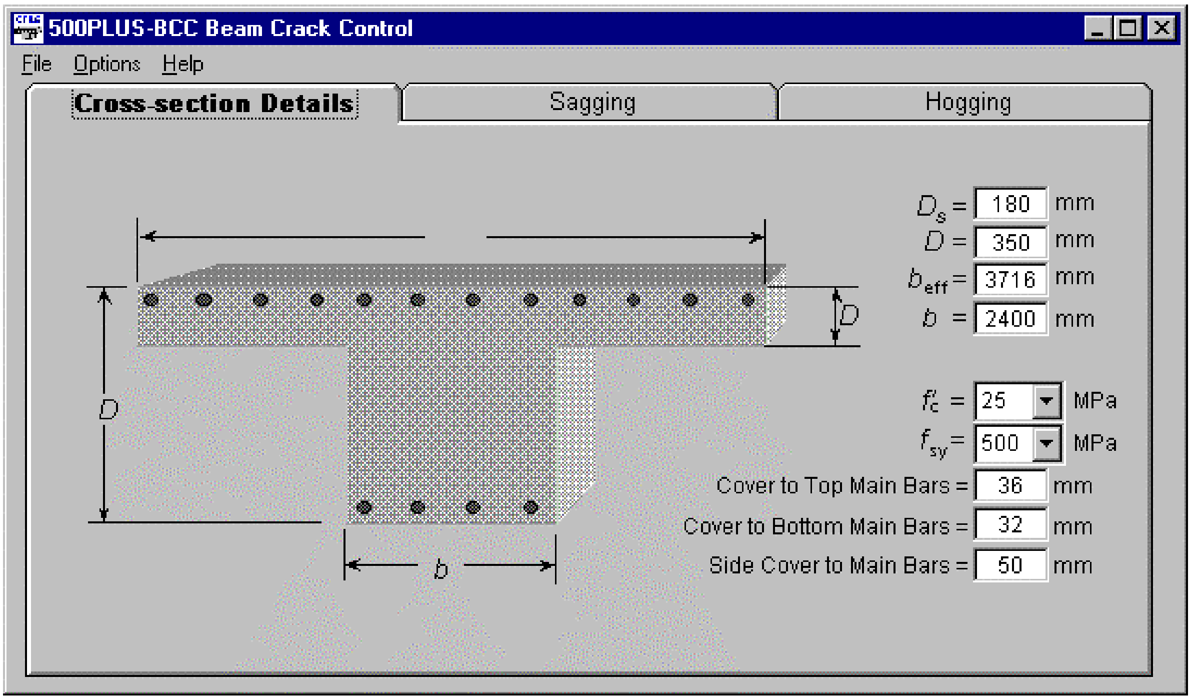
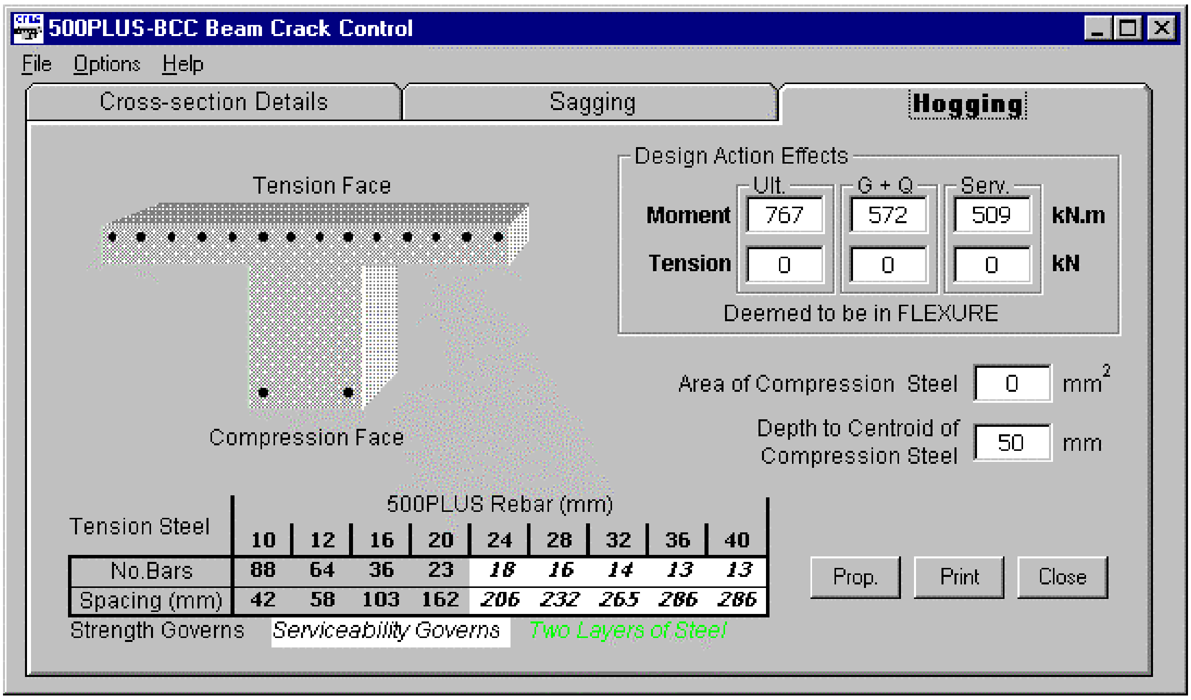
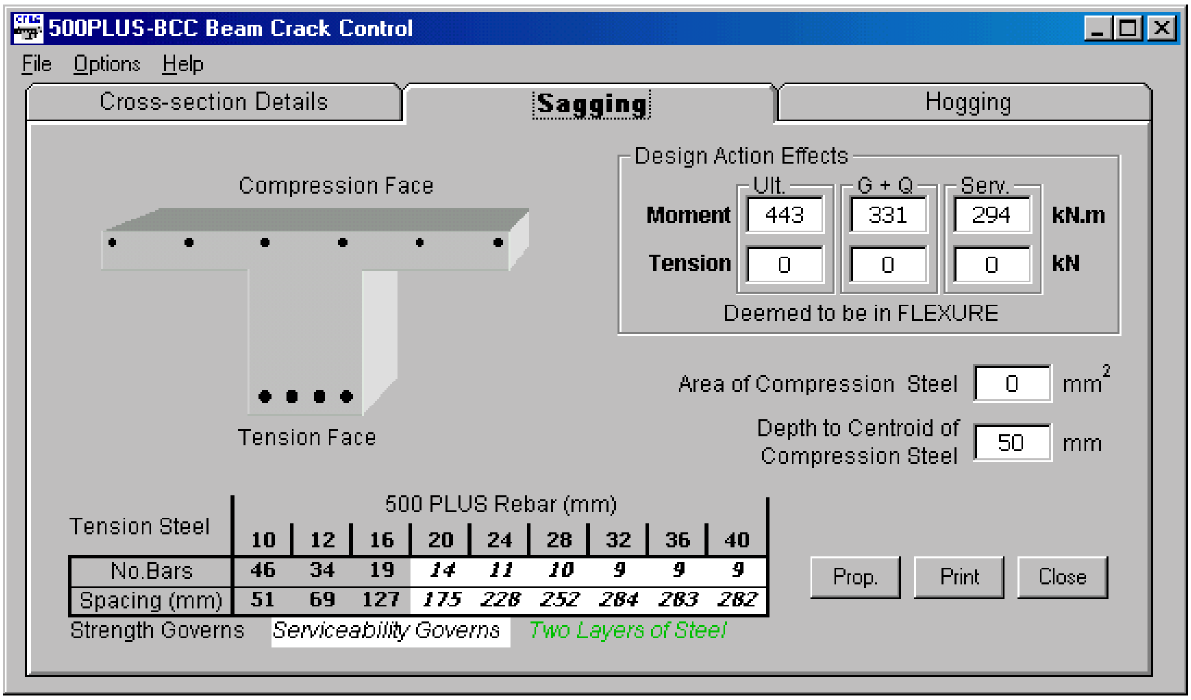


Figure 7.3 500PLUS-BCC[TM] – Cross-section Details (Example 2)
(Note: f'_c=32 MPa for the compactus load case.)



Tension Steel	500PLUS Rebar (mm)								
	10	12	16	20	24	28	32	36	40
No.Bars	88	64	36	23	18	16	14	13	13
Spacing (mm)	42	58	103	162	206	232	265	286	286

Figure 7.4 500PLUS-BCC™ Run for Case *Q*=4.0 kPa – No Moment Redistribution (Hogging)



Tension Steel	500 PLUS Rebar (mm)								
	10	12	16	20	24	28	32	36	40
No.Bars	46	34	19	14	11	10	9	9	9
Spacing (mm)	51	69	127	175	228	252	284	283	282

Figure 7.5 500PLUS-BCC™ Run for Case *Q*=4.0 kPa – No Moment Redistribution (Sagging)

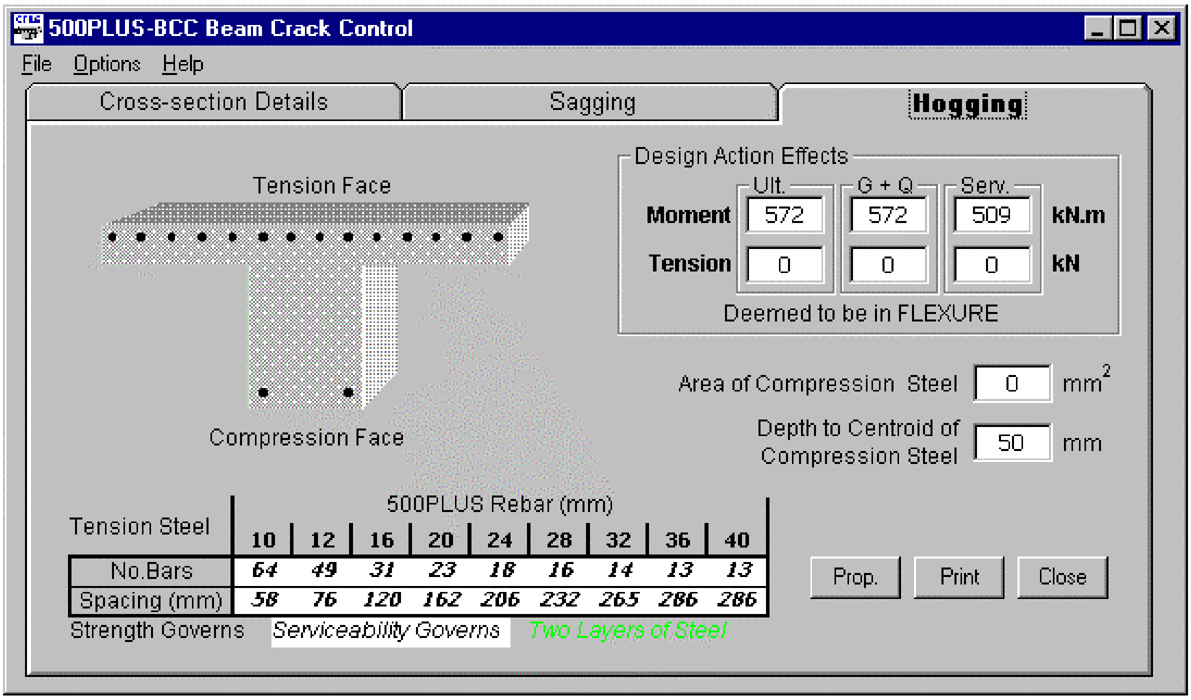


Figure 7.6 500PLUS-BCC™ Run for Case *Q*=4.0 kPa – Moment Redistribution (Hogging)
(Note: 500PLUS-BCC won't allow $M^* < M^*_{s.1}$ or M^*_s, so M^* has been made equal to 572 kNm.)

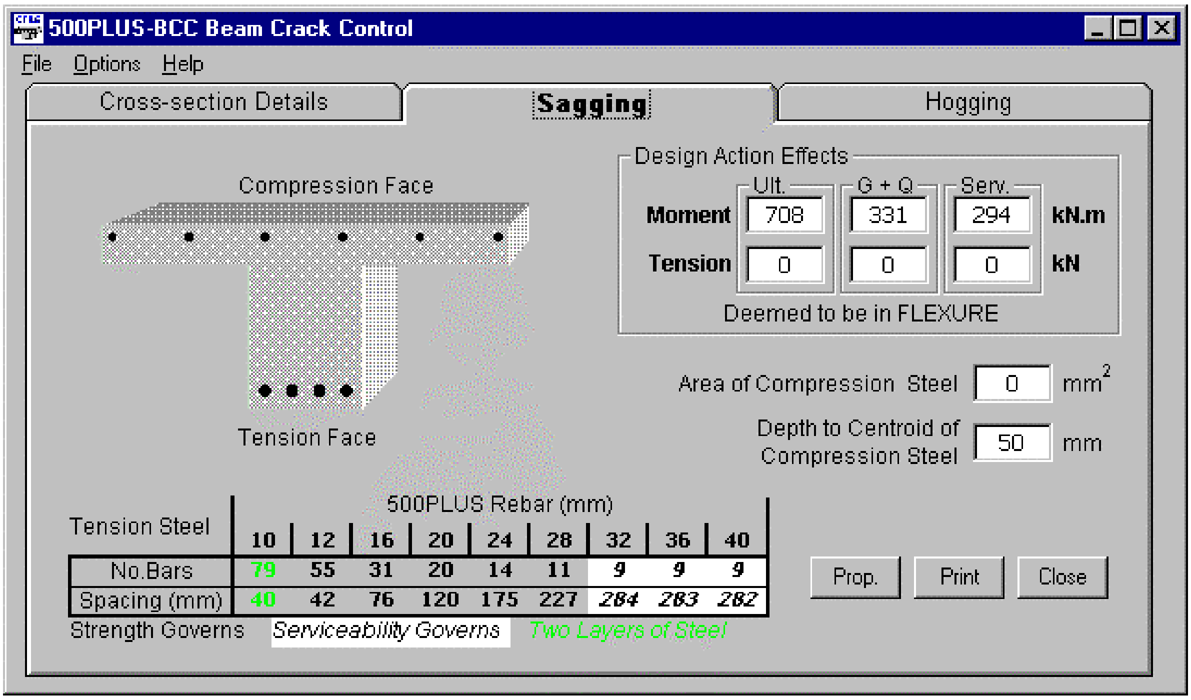


Figure 7.7 500PLUS-BCC™ Run for Case *Q*=4.0 kPa – Moment Redistribution (Sagging)

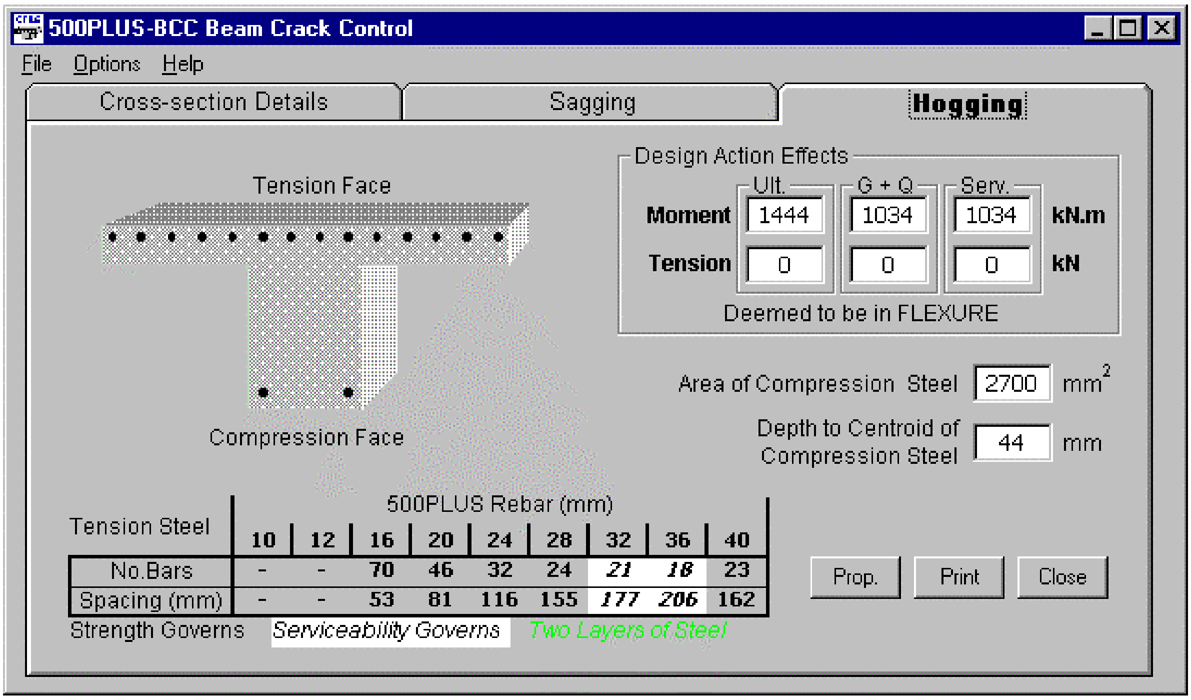


Figure 7.8 500PLUS-BCC™ Run for Case *Q*=10.0 kPa – No Moment Redistribution (Hogging)

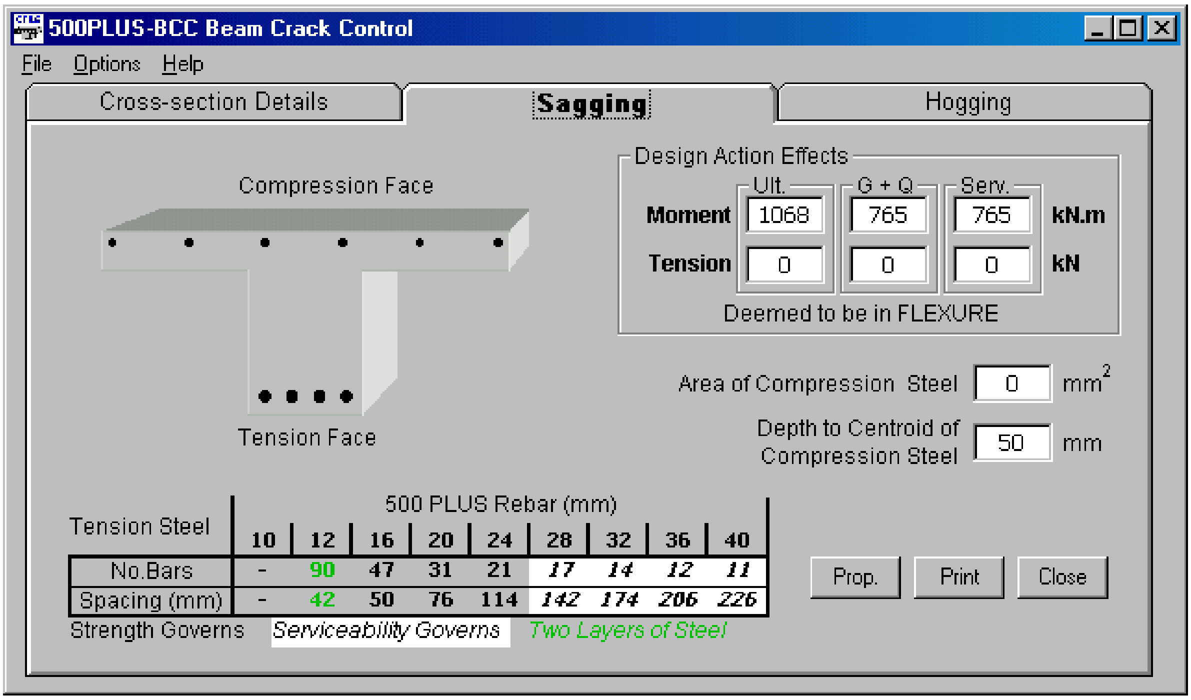
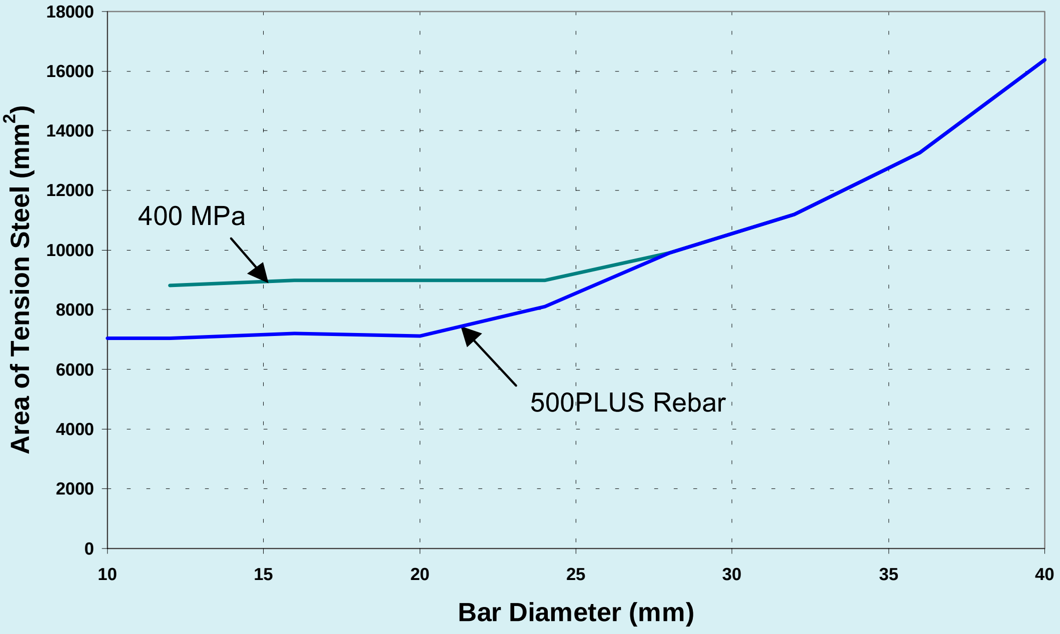
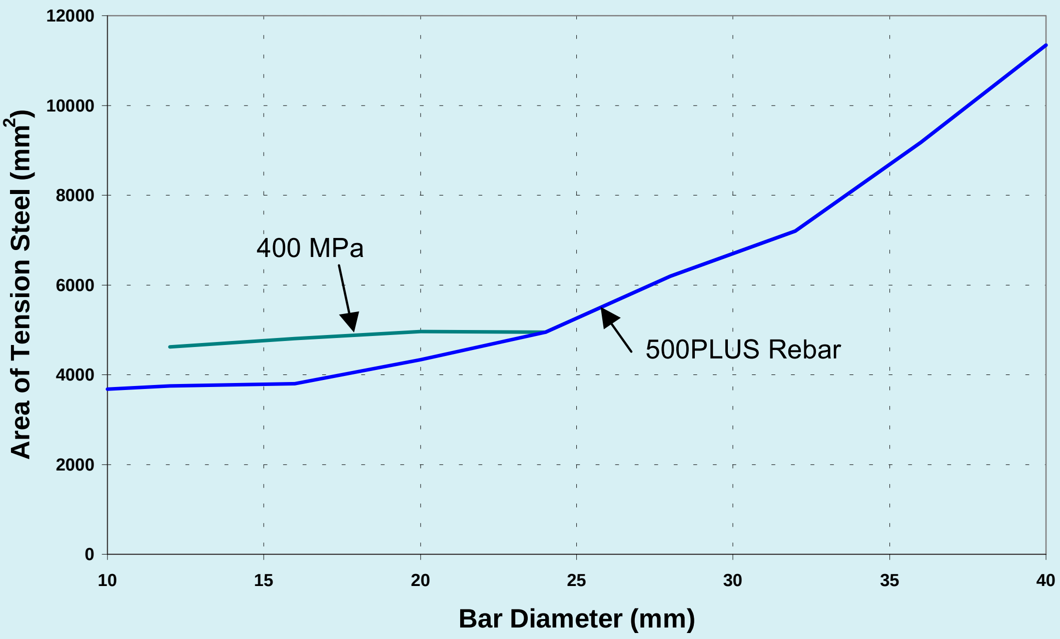


Figure 7.9 500PLUS-BCC™ Run for Case *Q*=10.0 kPa – No Moment Redistribution (Sagging)

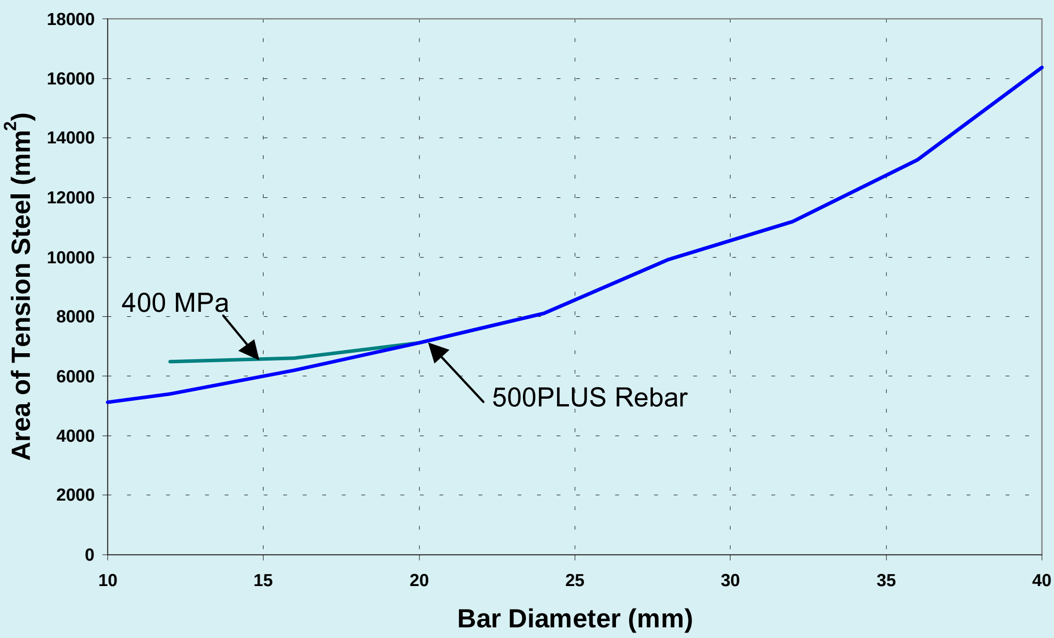


(a) Critical negative moment section

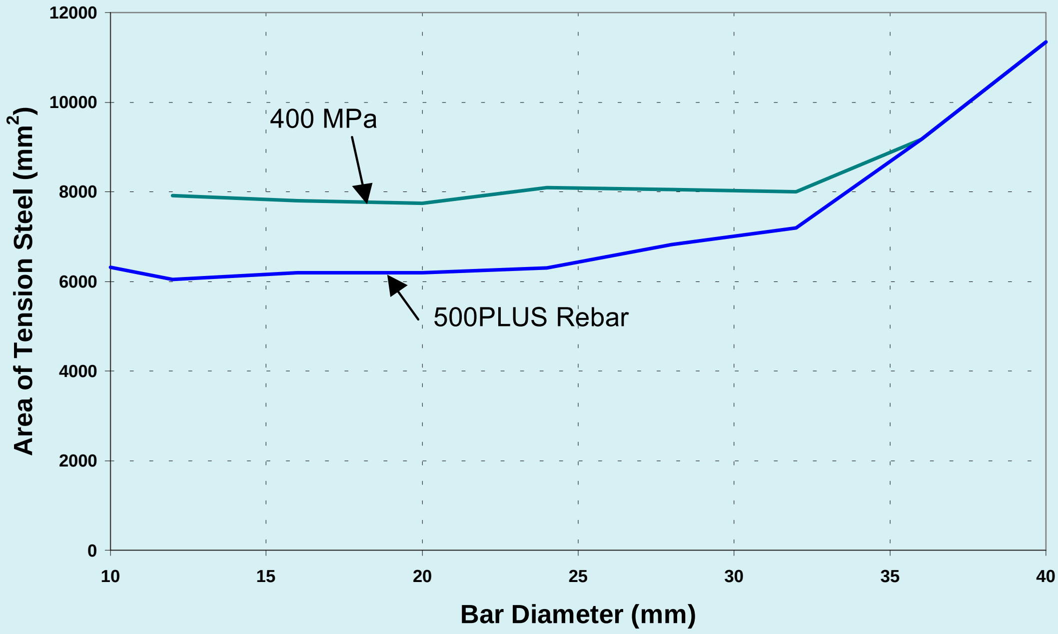


(b) Critical positive moment section

Figure 7.10 Effect of Bar Diameter and Steel Grade for Case *Q*=4.0 kPa – No Moment Redistribution

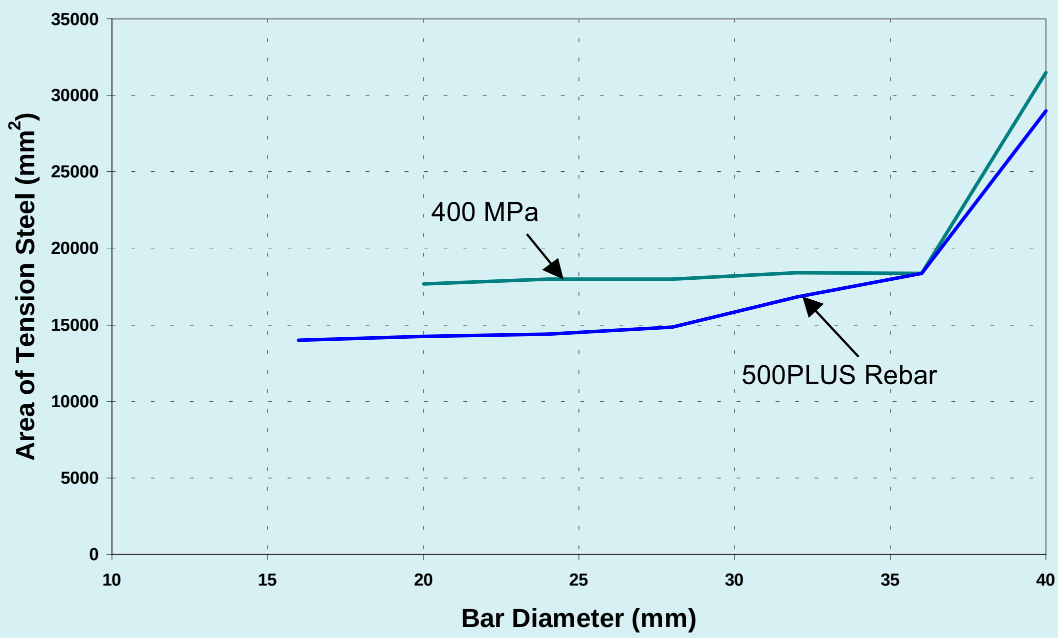


(a) Critical negative moment section

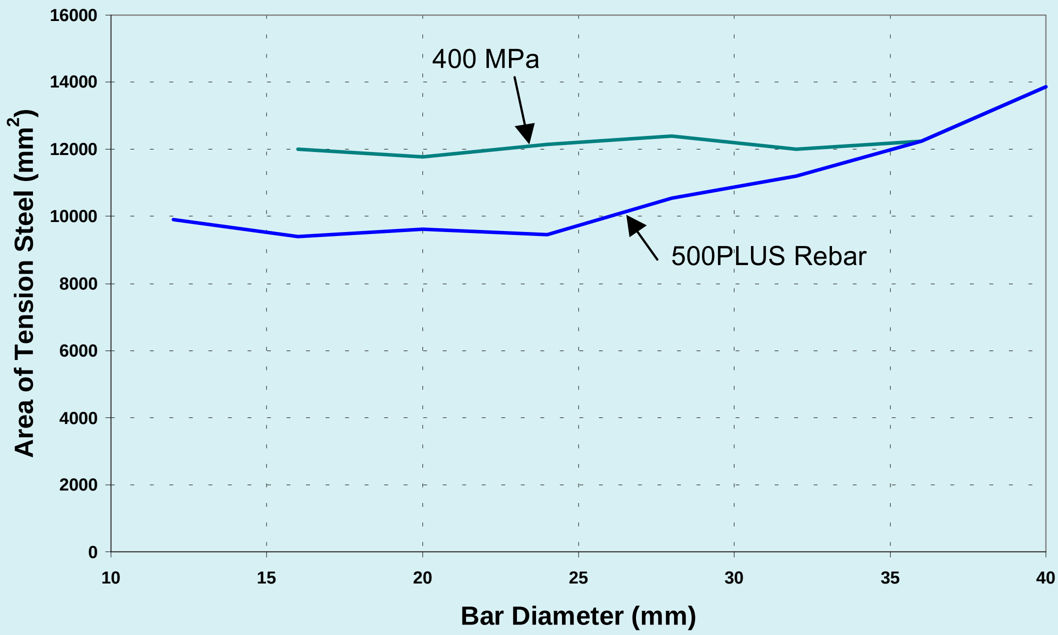


(b) Critical positive moment section

Figure 7.11 Effect of Bar Diameter and Steel Grade for Case *Q*=4.0 kPa – Moment Redistribution



(a) Critical negative moment section



(b) Critical positive moment section

Figure 7.12 Effect of Bar Diameter and Steel Grade for

Case *Q*=10.0 kPa – No Moment Redistribution

7.4 Example 3 – Fully Restrained Beam Designed to AS 3600-2000

The case of a fully restrained beam will be considered. A lightly-loaded facade beam is cast between two rigid walls like in Fig. 3.4(e), and must be designed for crack control. Shrinkage of the concrete is the main action that must be taken into account, since the direct loads that arise principally from self-weight are small and can be ignored. In fact, for the purpose of this example, the beam will be considered non-structural, and therefore the minimum strength requirement of Clause 8.1.4.1 of AS 3600-2000 will be ignored.

Gilbert [12,21] has considered the development of cracking in fully restrained members subjected to direct tension due to shrinkage. He uses as an example a 150 mm thick slab built-in at both ends. The situation is the same in principle as that shown in Fig. 3.1. He shows how to calculate the width of the first crack immediately it forms when the tensile strength of the concrete is reached. He also calculates the final spacing and width of the cracks that progressively form as the concrete continues to shrink with time. He considers both of the cases shown in Figs 3.1(a) and (b), viz.: “crack control” with enough steel to keep the final crack width to about 0.3 mm; and “no crack control” when the reinforcement has insufficient tensile capacity and yields. The first case of “crack control” is of interest here. The long-term (final) crack width is also of primary concern.

Exactly the same example Gilbert developed will be considered in relation to AS 3600-2000, except that the slab will be treated as a beam. The beam will be assumed to be 5.0 metres long, 1200 mm deep and 150 mm thick. It contains 12 mm diameter bars at 300 mm centres placed vertically in both sides giving A_{st}=750 mm^2/m. The side cover to these main bars is 50 mm, and f_t=2.0 MPa.

By considering exactly the same problem, the design action effects calculated by Gilbert can be used. These are summarised as follows, while the reader is referred to references [12] and [21] for the calculation procedure:

(a) Restraining force immediately after first cracking, T_s^* =161.3 kN/m.

(b) Final restraining force (long-term) with a fully-developed crack pattern, T_s^* =240.9 kN/m.

Although using different formulae to estimate crack width than presented herein, Gilbert calculates w=0.20 mm and 0.31 mm for each of these cases, respectively. The corresponding maximum tensile stress in the steel bars at the cracked section/s is 215 and 321 MPa, respectively.

It should be noted that the calculated value of the restraining force immediately after first cracking is directly proportional to the tensile strength of the concrete, f_t. Therefore, using a value of f_t=3.0 MPa in design, as required by AS 3600-2000, increases T_s^* at first cracking to 242 kN/m (=3.0/2.0×161.3). The final restraining force would also increase in this case, but not proportionally.

If one now considers the requirements proposed for inclusion in AS 3600-2000, Eq. 5.3(3) gives:

$$A_{st.min} = 3\, k_s A_{ct}/f_s$$

where –

k_s = 0.8, i.e. a tension state is assumed;

A_{ct} = 150×1000 =150,000 mm^2/m, noting that as explained in Section 3.5.3, the area of the tension steel should not be subtracted from the gross area; and

f_s = 330 MPa from Table 8.6.1(A) of AS 3600-2000 for d_b =12 mm.

This gives $A_{st.min}$ = 1090>750 mm^2/m, so AS 3600-2000 would require more steel, i.e. 12 mm diameter bars at 200 mm instead of 300 mm centres. This difference is directly due to the different values of f_t assumed in the calculations. Assuming f_t =2.0 MPa gives $A_{st.min}$ = 727≈750 mm^2/m, and then there is close agreement between both designs. Moreover, when the final restraining force acts, as stated above the tensile stress in the steel equals 321 MPa for f_t=2.0 MPa. This value of stress is close to 330 MPa, the maximum value allowed in AS 3600-2000, and 0.3 mm wide cracks would be expected as predicted by Gilbert. However, for f_t=3.0 MPa the larger steel area (12 mm bars at 200 mm centres) would also be required to keep the long-term steel stress below 330 MPa.

It should be noted that the yield strength of the reinforcement has not been a design factor in this particular example, because the minimum strength requirement (Eq. 5.3(3)) has governed the design. Therefore, the reinforcement required is the same irrespective of whether f_{sy}=400 or 500 MPa. However, this will not always be the case, and benefits can result using

500PLUS Rebar in situations like those examined in this example. Yielding is avoided in the example, i.e. in accordance with the design rules in Section 5.3 (also see Section 3.7.5), the maximum stress allowed equals f_{sy}. The results obtained from running program 500PLUS-BCC for this example are shown for interest in Figs. 7.13 and 7.14. (It can be seen in Fig. 7.13, that the cross-section is shown as a rectangle rather than a T-section. This occurs whenever D_s is assigned equal to zero.)

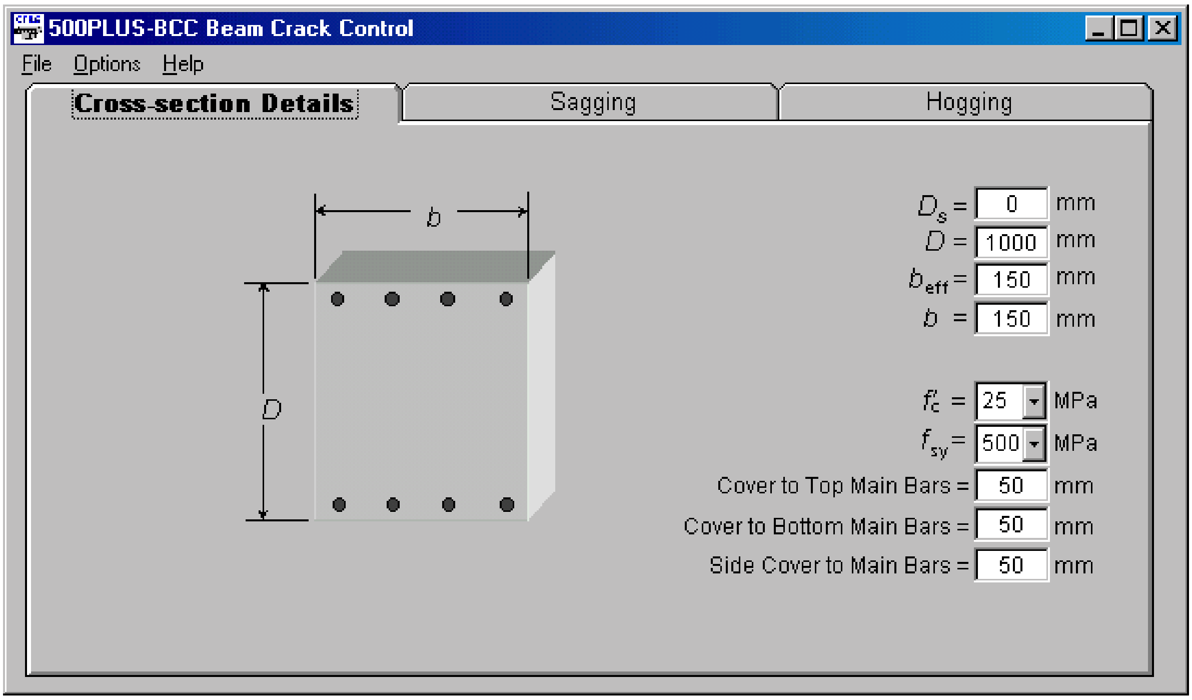


Figure 7.13 500PLUS-BCC™ – Cross-section Details (Example 3)

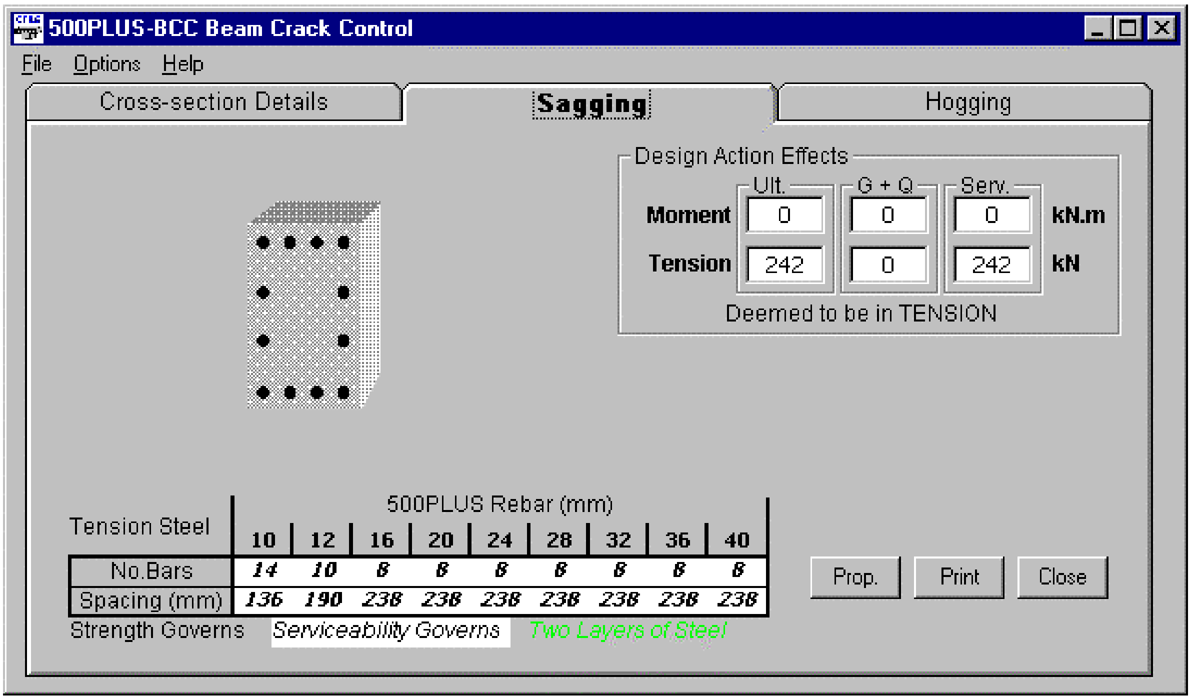


Figure 7.14 500PLUS-BCC™ Run for Tension Case (Example 3)

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APPENDIX A

REFERENCED AUSTRALIAN STANDARDS

REFERENCE NO.	TITLE
AS 1170.1-1989	Minimum Design Loads on Structures (known as the SAA Loading Code), Part 1: Dead and Live Loads and Load Combinations
AS 1302-1991	Steel Reinforcing Bars for Concrete
AS 1303-1991	Steel Reinforcing Wire for Concrete
AS 3600-1994	Concrete Structures
AS 3600/Amdt 1/1996-08-05	Amendment No. 1 to AS 3600-1994 Concrete Structures, August, 1996
AS 3600 Supp1-1994	Concrete Structures – Commentary
AS 3600/Amdt 1/1996-12-05	Amendment No. 1 to AS 3600-1994 Concrete Structures – Commentary, December, 1996
DR 99193 CP	Combined Postal Ballot/Draft for Public Comment Australian Standard, Amendment 2 to AS 3600-1994 Concrete Structures, Issued 1 May, 1999
AS 3600-2000[5]	Concrete Structures (including Amendments Nos 1 & 2)

[5] This Standard is yet to be published.

APPENDIX B

NOTATION

The notation used in this booklet has been taken from AS 3600-1994 when appropriate.

Latin Letters

a_s	length of span support (see Fig. C5.1)
A_b	cross-sectional area of a steel bar
A_c	cross-sectional area of concrete (steel excluded)
$A_{c.eff}$	effective cross-sectional area of concrete in tension that surrounds the tension reinforcement
A_{ct}	cross-sectional area of concrete in the tensile zone assuming the section is uncracked
A_g	gross cross-sectional area of concrete
$A_{s.bot}$	cross-sectional area of steel in bottom face
$A_{s.top}$	cross-sectional area of steel in top face
A_{sc}	cross-sectional area of compression steel
A_{st}	cross-sectional area of tension steel
$A_{st.min}$	minimum area of reinforcement permitted in tensile zone
b	beam width
b_{eff}	beam flange effective width calculated in accordance with Clause 8.8.2 of AS 3600-1994
b_t	mean width of tension zone (see Section 3.7.3)
b_w	width of beam web
c	concrete cover
d	effective depth of reinforcement at a section in bending
d_b	nominal diameter of reinforcing bar
d_{bot}	effective depth of bottom reinforcement at a section in bending
$d_{c.eff}$	depth of effective tension area of concrete (see Fig. 3.5)
d_n	depth of elastic neutral axis below compressive face at a cracked section
d_s	diameter of stirrup or fitment
d_{sc}	depth of centroid of compression reinforcement below compression face
d_{top}	effective depth of top reinforcement at a section in bending
D	overall depth of beam
D_s	overall depth of beam flange (normally part of a slab)
e	eccentricity of prestressing force (see Eq. 5.3(1))
e_T	eccentricity of design tensile force (see Section 3.6.1)
E_c	modulus of elasticity of concrete, calculated in accordance with Clause 6.1.2 of AS 3600-1994 for design
E_s	modulus of elasticity of steel reinforcement (=200 GPa)
f'_c	characteristic compressive cylinder strength of concrete at 28 days

f'_{cf}	characteristic flexural tensile strength of concrete calculated in accordance with Clause 6.1.1.2 of AS 3600-1994
f_{cm}	mean compressive strength of concrete in accordance with AS 3600-1994 (see p. 149 of AS 3600 Commentary)
f_{ct}	tensile stress in concrete (see Fig. 3.2)
f_s	tensile stress in reinforcement (as a general term), or (in AS 3600-2000) maximum tensile stress permitted in the reinforcement immediately after the formation of a crack (see Eqs 3.5.3(1) and 5.3(3))
f_{scr}	tensile stress in reinforcement at a cracked section
$f_{scr.1}$	tensile stress in reinforcement at a cracked section, calculated with ψ_s=1.0
f_{sr}	stress in tension steel that just causes the tensile strength of the concrete to be reached (see Eq. 3.5.2(5))
f_{sy}	yield strength of steel reinforcement
f_t	tensile strength of concrete (mean value in Eurocode 2 – see Eq. 3.5.3(1)), assumed to equal 3.0 MPa during design for states of either tension or flexure when using Eq. 5.3(3)
G	total dead load (including G_{cs} and G_{sup})
G_{cs}	dead load of concrete and reinforcing steel supported by beam
G_{sup}	superimposed dead load supported by beam
I_{av}	average second moment of area calculated in accordance with AS 3600-1994
I_{cr}	second moment of area of a cracked section
I_{uncr}	second moment of area of an uncracked section
k	ratio of depth of elastic neutral axis, d_n, to effective depth, d, at a cracked section (see Figs 5.7, 5.8 and 5.9)
k_u	neutral axis parameter (strength limit state)
$\overline{k}$	ratio of depth of elastic neutral axis, x, to overall depth, D, at an uncracked section (see Figs 5.3 and 5.4)
k_s	a coefficient that takes into account the shape of the stress distribution within the section immediately prior to cracking, as well as the effect of non-uniform self-equilibrating stresses (see Eq. 5.3(3)) (=k_3 x k_4)
k_1	a factor that takes account of the bar bond properties (see Eq. 3.5.2(4)), or a coefficient used in the equation to calculate I_{av}
k_2	a factor that takes account of the stress distribution (see Eq. 3.5.2(4))
k_3	a factor that allows for the effect of non-uniform self-equilibrating stresses (see Eq. 3.5.3(1))
k_4	a factor that takes account of the stress distribution immediately prior to cracking (see Eq. 3.5.3(1))
l_{tr}	transfer length
M_{cr}	cracking moment, calculated ignoring effects of concrete shrinkage as per AS 3600-1994 (In AS 3600-2000, it is proposed that this term will be redefined to include the effects of concrete shrinkage, which leads to a reduced, more realistic value of flexural stiffness for deflection calculations – see DR 99193 CP)
M^*	design bending moment at strength limit state
M^*_s	design bending moment at serviceability limit state
$M^*_{s.1}$	design bending moment at serviceability limit state, calculated with ψ_s=1.0

M_{sy}	moment capacity at a cracked section of a reinforced-concrete beam assuming the steel has yielded, i.e. under-reinforced
M_{uo}	nominal or ultimate strength in bending
$(M_{uo})_{min}$	minimum nominal strength in bending permitted at critical sections (see Eq. 5.3(1))
n	modular ratio ($=E_s/E_c$)
n_b	number of bars in a bundle (see Section 3.7.2)
p	reinforcement ratio for bending of a cracked section ($=A_{st}/bd$)
p_{min}	minimum reinforcement ratio (see Fig. 3.6)
$\overline{p}$	reinforcement ratio for bending of an uncracked section ($=A_{st}/bD$)
p_s	reinforcement ratio for tension ($=A_{st}/A_c$)
p_r	effective reinforcement ratio ($=A_{st}/A_{c.eff}$)
P	prestressing force (see Eq. 5.3(1))
Q	live load
Q_{com}	live load in compactus area
Q_{off}	live load in office area
R_s	corner radius of stirrup or fitment
s_b	bar spacing (see Eq. 3.6.2(7)
s_{cr}	crack spacing
s_{crm}	average final crack spacing
$s_{cr.min}$	minimum crack spacing
$s_{cr.max}$	maximum crack spacing
T^*	design tensile force at strength limit state (Note: In AS 3600, this symbol is used for the design torsional moment.)
T_b	tensile force in steel bar at a crack
T_b'	tensile force in steel bar at end of transition length, where full bond exists
T_{cr}	tensile force at first cracking (i.e. immediately prior to cracking), equal to the sum of the tensile forces in the steel reinforcement and the concrete
T_s^*	design tensile force at serviceability limit state, assumed to be located at the centroid of the uncracked section
$T_{s.1}^*$	design tensile force at serviceability limit state, assumed to be located at the centroid of the uncracked section, and calculated with ψ_s=1.0
w	crack width
w_k	design crack width (characteristic value)
w_{max}	maximum crack width
x	depth of elastic neutral axis below top face of beam at an uncracked section (see Figs 5.3 and 5.4)
X	parameter used to calculate elastic section properties
$\overline{X}$	parameter used to calculate elastic section properties
Y	parameter used to calculate elastic section properties
$\overline{Y}$	parameter used to calculate elastic section properties

z	lever arm of internal force couple (see Eq. 3.6.1(1) and Fig. 5.7)
Z	section modulus of uncracked section, referred to the extreme fibre at which flexural cracking occurs (see Eq. 5.3(1))
Z_c	section modulus of uncracked section, referred to the extreme compression fibre at which flexural cracking occurs (see Eq. 3.6.1(2))
Z_t	section modulus of uncracked section, referred to the extreme tension fibre at which flexural cracking occurs (see Eq. 3.6.1(2)) (=Z)

Greek Letters

β	a factor that relates the mean crack width in tests to the design value (see Eq. 3.5.2(1))
β_1	a factor that accounts for the bond properties of the reinforcement (see Eq. 3.5.2(5))
β_2	a factor that accounts for repeated stressing of the reinforcement (see Eq. 3.5.2(5))
χ	ratio of lever arm to internal force couple, to effective depth, d
Δ	support movement
ε_c	concrete strain
ε_{cm}	average or mean concrete strain over transition length l_{tr}
ε_{cs}	free shrinkage strain of concrete
ε_s	steel strain
ε_{sm}	average or mean steel strain over transition length l_{tr} , or average difference in strain between steel and concrete (see Eqs 3.5.2(1) and 3.5.2(5))
ϕ	capacity factor (see Table 2.3 of AS 3600-1994)
η	degree of moment redistribution (see Eq. 4.1(2))
κ	parameter used to calculate second moment of area
λ_p	load factor to account for stage construction effects, i.e. propping (see Fig. C3.1)
$\lambda_{p.min}$	minimum value of λ_p (see Fig. C3.1)
Σ_o	bar perimeter
ρ_c	density of concrete (excluding allowance for steel reinforcement)
τ	bond stress
τ_m	mean bond stress
ξ	parameter that accounts for moment gradient effects (see Eq. 3.3.3.2(2))
ψ_l	long-term load factor (see AS 1170.1)
ψ_s	short-term load factor (see AS 1170.1)

APPENDIX C

CALCULATION OF DESIGN BENDING MOMENTS UNDER SERVICE LOADS FOR CRACK CONTROL DESIGN

C1 Introduction

Linear elastic analysis is the most useful way of calculating the design bending moments under service loads for crack control design.

C2 Linear Elastic Analysis

Clause 7.6 *Linear Elastic Analysis* of AS 3600-1994 addresses the use of linear elastic analysis to determine the design action effects in a structure for both strength and serviceability design.

Elastic analysis is normally based on the stiffness of the uncracked (gross) sections for both strength and serviceability limit states. Clause 7.6.5 *Stiffness* of AS 3600-1994 permits this, or, if it is suspected that cracking may have a significant effect, then a more realistic analysis taking cracking into account is allowed. This latter possibility would be ignored for the serviceability design of normal building structures. Analysis based on uncracked sections implies that moment redistribution will be ignored in design.

C3 Pattern Loading

The requirement to consider pattern loading is particularly relevant to design for serviceability.

It has been proposed in DR 99193 CP (see Appendix A, Referenced Australian Standards), that Clause 7.6.4 *Arrangement of vertical loads for buildings* of AS 3600-2000 will be re-titled to refer to vertical live loads, since there was some ambiguity with regard to the effect of prestress. However, for reinforced-concrete structures there is no change proposed to the requirements for pattern live loads.

Two approaches that can be used to satisfy the intent of Clause 7.6.4, and take account of the effect of pattern loading when appropriate for serviceability design, are as follows:

(a) provided the methods of analysis for the two different limit states are consistent with each other, e.g. uncracked sections and elastic behaviour are assumed in both cases, then the design serviceability moment, M_s^*, at a critical section can be calculated by factoring the bending moment for the strength limit state (see Section 7.3, Eq. 7.3(1)); otherwise/or

(b) the design bending moment envelope for serviceability is calculated separately using the loading patterns shown in Fig. C3.1.

C4 Actions for Serviceability Design

C4.1 Loads

The design loads for crack control design shall be determined in accordance with the requirements of Section 5.3:

(a) by taking the appropriate combination of factored loads for short-term effects given in AS 1170.1 and using the appropriate value for the short-term load factor, ψ_s; noting that

(b) it is also necessary, when designing against the tension reinforcement from possibly yielding (i.e. $f_s \leq 0.8 f_{sy}$) to assign a value of unity to the short-term load factor (see Section 5.3, proposed Clause 8.6.1(d) of AS 3600-2000).

Note: It is acceptable to scale the design serviceability moments calculated according to (a) by $1.0/\psi_s$, in order to calculate the design serviceability moments for (b).

C4.2 Other Actions

Consideration should be given during design to the possible effects of the other actions listed in Clause 3.1.3 of AS 3600-1994. Many of these actions are often a major cause of cracking in reinforced-concrete structures, and yet it is common to completely overlook them during design. Although this will require more detailed design calculations to be performed, it is wise to quantify their effect if cracking is to be controlled with any reasonable degree of reliability.

Settlement or rotation of supports can significantly change the distribution of bending moments under serviceability conditions, and should not be overlooked when the soil conditions are conducive to differential foundation movements occurring.

Shrinkage and temperature movements are other major causes of cracking when deformation is restrained.

C5 Critical Sections for Negative Design Bending Moments

During elastic analysis, the walls or columns that rigidly support a reinforced-concrete beam are normally modelled as knife-edges. Account should be taken of the finite width of these supports when calculating the maximum negative design bending moments used in crack control design of the beam, where it is continuous over internal supports. This issue is addressed in Clause 7.6.10 of AS 3600-1994 and Clause 2.5.3.3(4) of Eurocode 2.

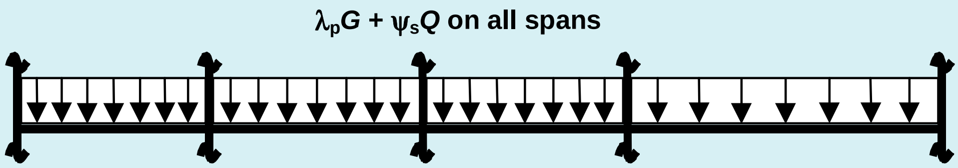
In accordance with Clause 7.6.10 of AS 3600-1994, at internal supports the critical section for negative bending should be taken at 0.35 times the overall width of the support, $2a_s$, where a_s is the length of the span support, measured out from the support centreline (see Fig. C5.1(a)). (In Eurocode 2, the requirement is stated differently but is equivalent to a factor of 0.25 being used instead of 0.35, and is therefore more conservative.) However, when the support is wider than half the overall depth of the beam, D, consideration should be given to modelling it as two knife-edge supports located 0.8 times the width of the support, $2a_s$, apart in order to calculate the peak negative and positive moments (see Fig. C5.1(b)).

C6 Construction Sequence

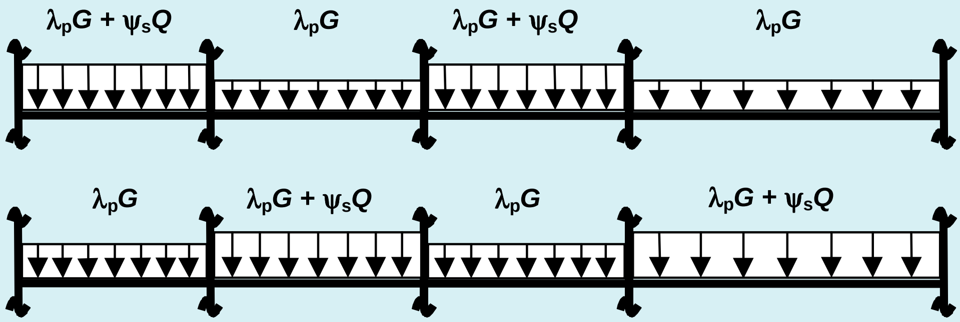
A new paragraph is proposed in DR 99193 CP for inclusion in Clause 7.6.2 of AS 3600-2000, viz. “Where the construction involves prefabricated elements supporting insitu construction, consideration in the analysis shall be given to the proposed construction sequence and the degree of propping of the prefabricated elements during construction.”. This is particularly relevant when designing for serviceability.

It is normal to assume in conventional reinforced-concrete construction that the floor members are constructed “fully-propped”, i.e. the removable formwork system supports the weight of the wet concrete and reinforcement while the concrete hardens. This way, the final structure must be designed for serviceability assuming the full effects of the dead loads and serviceability live loads.

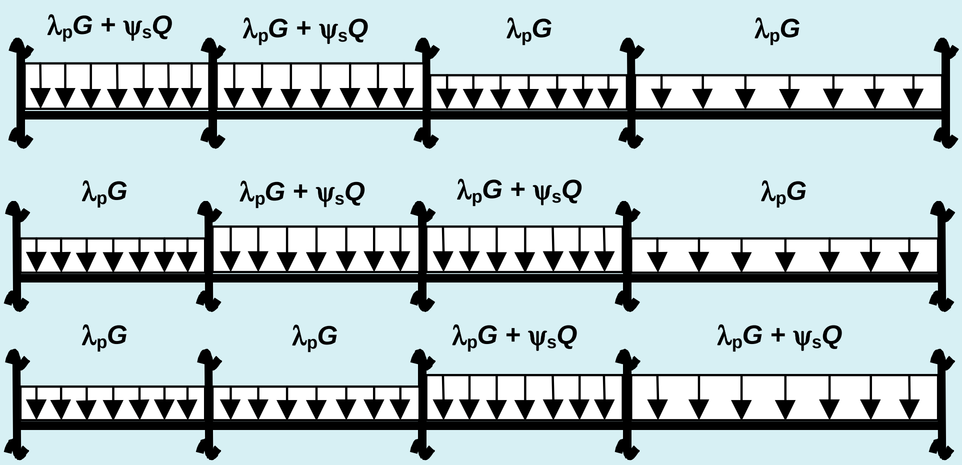
In “unpropped” or “partially-propped” construction such as occurs when prefabricated elements are used (e.g. precast beams with an insitu topping, possibly with a mid-span prop), then the hardened concrete will be less likely to be cracked compared with “fully-propped” construction.



(a) All spans loaded



(b) Alternate spans loaded (iff Q>0.75G)



(c) Adjacent spans loaded (iff Q>0.75G)

Note: Load factor λ_p accounts for the effects of propping, such that $\lambda_{p.min} \leq \lambda_p \leq 1.0$, where $\lambda_{p.min} = G_{sup}/G$ and G_{sup} is the superimposed dead load.

For example, for conventional "fully-propped" construction, λ_p = 1.0, while in unpropped construction λ_p = 0.0 if G_{sup}=0 and $G = G_{cs}$ where G_{cs} is the dead load of the concrete and reinforcement in the beam.

Figure C3.1 Pattern Loading Cases for Serviceability Analysis

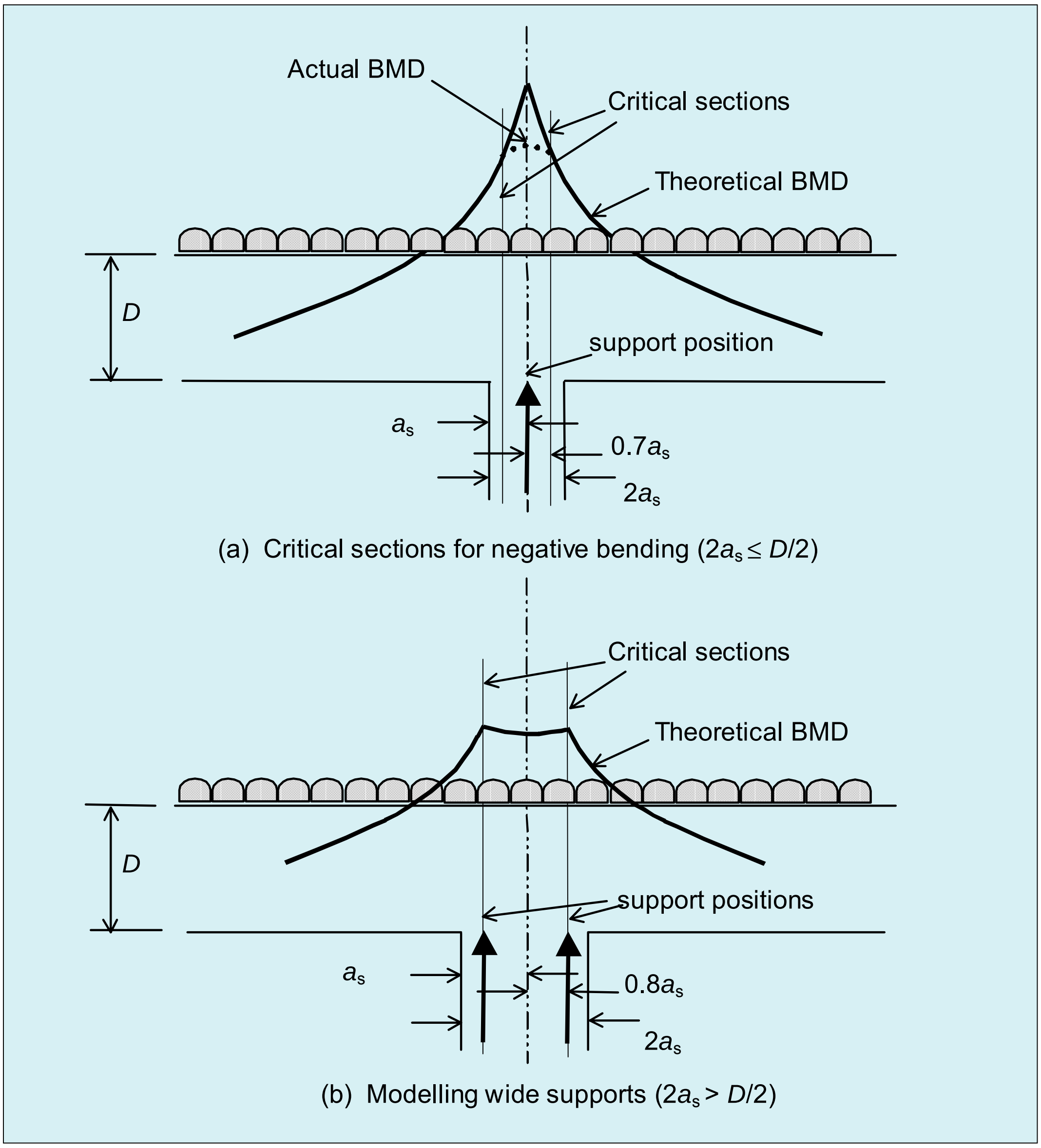


Figure C5.1 Critical Sections and Modelling of Supports for Calculation of Peak Negative Design Bending Moments at Internal Supports

APPENDIX D

EXTRACTS FROM DR 99193 CP – DRAFT AMENDMENT NO. 2 TO AS 3600-1994

D1 Introduction

This appendix contains extracts from the draft for public comment DR 99193 CP, an early version of Amendment No. 2 to AS 3600-1994. The proposed design rules presented in Section 5.3 are different to the public comment draft (which must not be used for design purposes). Clauses in Section 5.3 that are not contained in Paragraph D2 (viz. Clauses 2.4.4, 8.1.7, 8.6.3 and 8.6.4) have been taken directly from AS 3600-1994, and remain unchanged (except Clause 8.6.3 that has had minor references to Y bars removed).

Standards Australia intends to revise AS 3600-1994 by incorporating into its body, Amendment No. 1 (1996) and Amendment No. 2 (once finalised), and to publish it as AS 3600-2000.

D2 Draft Amendment No. 2 to AS 3600-1994

The information presented below must not be used for design purposes.

The changes that were proposed to the clauses in AS 3600-1994 and issued for public comment as DR 99193 CP, that are relevant to Section 5 of this design booklet, are as follows.

8.1.4 Minimum strength requirements

8.1.4.1 General Unless it can be demonstrated that the onset of cracking at any cross-section will not lead to sudden collapse of the member, the ultimate strength in bending, (M_{uo}) at critical sections shall not be less than $(M_{uo})_{min.}$ where–

$$(M_{uo})_{min.} = 1.2\,[Z(f'_{cf} + P/A_g) + Pe]$$

where

Z = the section modulus of the uncracked section, referred to the extreme fibre at which flexural cracking occurs

f'_{cf} = the characteristic flexural tensile strength of the concrete

e = the eccentricity of the prestressing force (P) is measured from the centroidal axis of the uncracked section

For rectangular reinforced concrete cross-sections, this requirement shall be deemed to be satisfied if minimum tensile reinforcement is provided such that –

$$A_{st}/bd \geq 0.22\,(D/d)^2\,f'_{cf}\,/\,f_{sy}$$

8.6.1 Crack control for tension and flexure in reinforced beams Cracking in reinforced beams subjected to flexure or tension shall be deemed to be controlled if the appropriate requirements in Items (a) and (b), and either Items (c) or (d) are satisfied. For the purpose of this Clause, the resultant action is considered to be flexure when the tensile stress distribution within the section prior to cracking is triangular with some part of the section in compression, or tension when the whole of the section is in tension.

(a) The minimum area of reinforcement in the tensile zone (A_{gt}) shall be taken as–

$$A_{gt} = 3\, k_s\, A_{ct}/f_s$$

where

k_s = a coefficient which takes into account the shape of the stress distribution within the section immediately prior to cracking, and equals 0.4 for flexure and 0.8 for tension

A_{ct} = the area of concrete in the tensile zone, being that part of the section in tension assuming the section is uncracked

f_s = the maximum tensile stress permitted in the reinforcement assuming the section is cracked, which for beams subjected to flexure is 0.5 times the yield strength of the reinforcement ($0.5f_{sy}$), and for beams subjected to tension the lesser of –

(i) the yield strength of the reinforcement (f_{sy}); and

(ii) the maximum steel stress given in Table 8.6.1(A) for the bar diameter used.

(b) The distance from the side or soffit of a beam to the centre of the nearest longitudinal bar shall not be greater than 100 mm. Bars with a diameter less than half the diameter of the largest bar in the cross-section shall be ignored.

(c) For beams subjected to tension, the nominal diameter (d_b) of the bars shall not exceed the appropriate value (d_b*) given in Table 8.6.1(A), such that the steel stress calculated using the load combination for serviceability design with long-term effects shall not exceed the maximum steel stress.

TABLE 8.6.1(A)

MAXIMUM BAR SIZE FOR TENSION OR FLEXURE

Maximum steel stress (MPa)	**Maximum nominal bar diameter, d_b* (mm)**
160	32
200	25
240	20
280	16
320	12
360	10
400	8
450	6

(d) For beams subjected to flexure, the nominal diameter of bars (d_b) in the tensile zone shall not exceed the appropriate value (d_b*) given in Table 8.6.1(A), such that the steel stress calculated using the load combination for serviceability design with long-term effects shall not exceed the maximum steel stress. Alternatively, the centre-to-centre spacing of adjacent parallel bars in the tensile zone, ignoring bars with a diameter less than half the diameter of the largest bar in the cross-section, shall not exceed the maximum spacing determined from Table 8.6.1(B).

TABLE 8.6.1(B)

MAXIMUM BAR SPACING FOR FLEXURE

Maximum steel stress (MPa)	Maximum centre-to-centre spacing (mm)
160	300
200	250
240	200
280	150
320	100
360	50

NOTE: Linear interpolation may be used

APPENDIX E

MAXIMUM STEEL STRESS AS A FUNCTION OF BAR DIAMETER OR BAR SPACING

E1 Introduction

A simple rectangular beam in bending is used to explain using the relationships for maximum tensile stress and bar diameter (Table 8.6.1(A) of AS 3600-2000) and maximum tensile stress and bar spacing (Table 8.6.1(B) of AS 3600-2000) defined in Section 5.3.

E2 Bar Diameter vs Bar Spacing

The rule proposed as Clause 8.6.1 of AS 3600-2000 in Section 5.3 allows a designer to choose a suitable bar spacing in the event that the tensile stress in the steel exceeds the limit given in Table 8.6.1(A) for a chosen bar diameter. Advantage was taken of this option in Part 2 of Example 1 in Section 7.2, when designing the tension reinforcement in the positive moment region of the beam shown in Fig. 7.2.

Consider the singly-reinforced rectangular beam shown in Fig. E2.1, where for simplicity the bars in the tension face are assumed to be equi-sized, and equi-spaced across the width of the beam. Also, the distance from the centre of the outer bars to each adjacent side of the beam is assumed to equal half the bar spacing, s_b. Therefore, it can be written that $p = A_{st}/(bd) = A_b/(s_b d)$ (see Eq. E2(3) below).

From the notes to Table 8.6.1(A), it can be written that:

$$f_s = -173\log_e(d_b) + 760 \text{ MPa} \qquad \text{E2(1)}$$

where f_s is the maximum tensile stress permitted in the reinforcement.

Similarly, from the note to Table 8.6.1(B), it can be written that:

$$f_s = -0.8\, s_b + 400 \text{ MPa} \qquad \text{E2(2)}$$

Two simple further equations can be written, viz.:

$$s_b = \frac{\pi}{4pd} d_b^2 \qquad \text{E2(3)}$$

and from Section 5.4,

$$\min . s_b = d_b + 30 \text{ mm} \qquad \text{E2(4)}$$

Substituting Eq. E2(3) into Eq. E2(2), and equating the right-hand side of this equation with that of Eq. E2(1) defines values of bar diameter, d_b, when the maximum tensile stress permitted by the two approaches is the same. An example of the type of solution this gives is shown in Fig. E2.1 for a beam with an effective depth, d =300 mm, which was chosen arbitrarily. (Points marked on the curve labelled Eq. E2(1)=Eq. E2(2) with the same value of p represent pairs of multiple solutions obtained when solving these equations.)

It should be clear from Fig. E2.1, that the bar spacing rule can only provide the designer with additional solutions (see crossed region), when the reinforcement ratio, p, lies above a lower value defined by the curve Eq. E2(1)=Eq. E2(2), which depends on the bar diameter, d_b. (For simplicity, upper and lower limits that apply to p for strength requirements have been omitted from Fig. E2.1, but could be included by a designer interested in using this approach.)

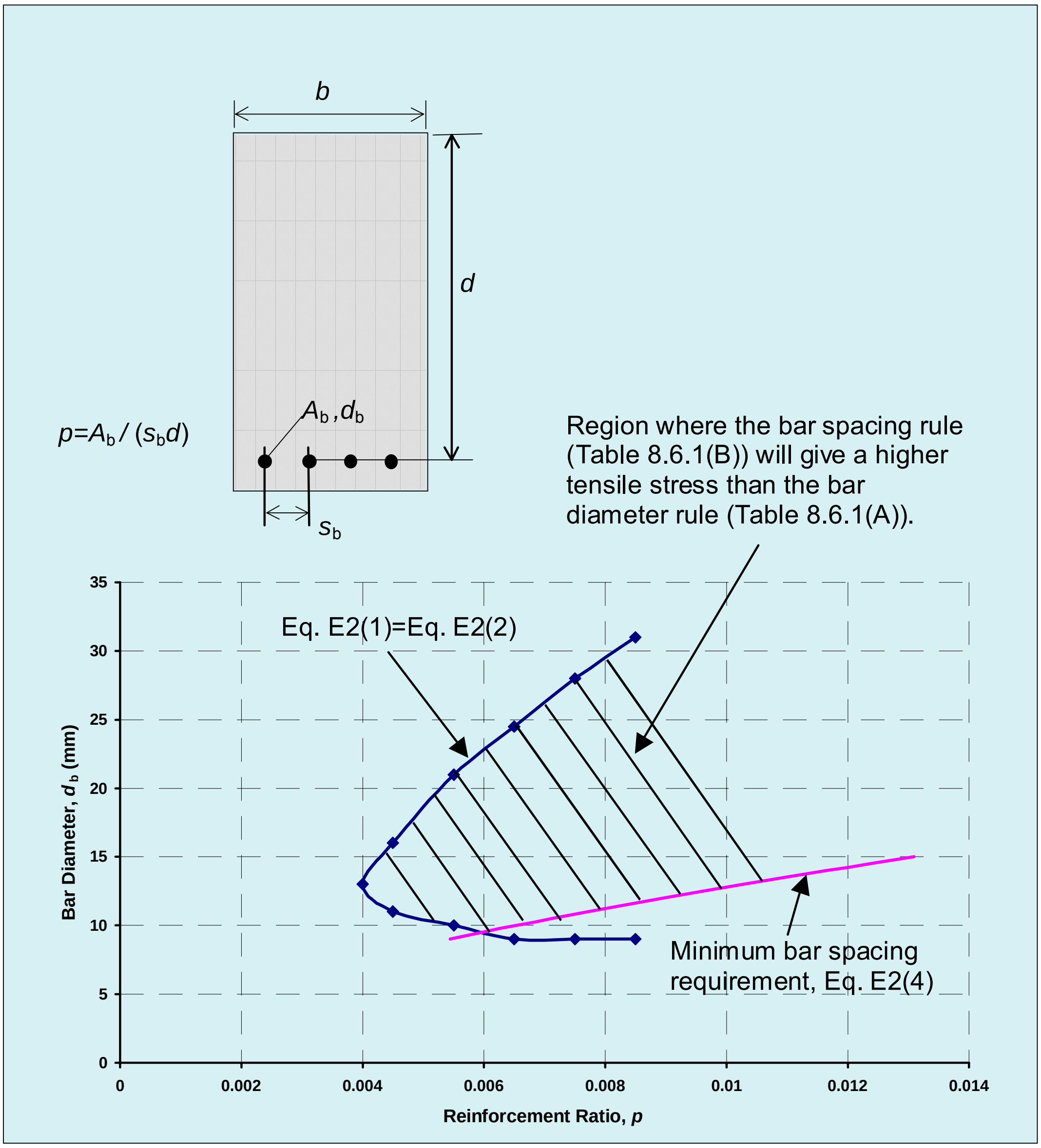


Figure E2.1 Example Beam showing Region of Applicability for Bar Spacing Rule